Francisco J. Tapiador, Ziad S. Haddad and Joe Turk

PROBABILISTIC RDSD

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Why a probabilistic approach?

Empirical, statistical fits to the gamma distribution are not Probability Distribution Functions (PDFs)

$$n(D)_{MP} = N_0 e^{-\Lambda D}$$

$$n(D)_{Ulb} = N_0 D^{\mu} e^{-\Lambda D}$$

Normalized RDSD

Empirical, statistical fits to the gamma function are not proper PDFs

$$n(D)_{Tes} = N_0^* F_\mu(D/D_m)$$

$$F_\mu(X) = \frac{\Gamma(4)}{4^4} \frac{(4+\mu)^{4+\mu}}{\Gamma(4+\mu)} X^\mu e^{-(4+\mu)X}$$

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What is a PDF, then?

- Empirical fits to gamma functions are not PDFs
- What is a PDF?

$$\left.\begin{aligned} p(D) \in [0,1] \\ \int p(D)dD = 1 \\ p(D_1 \bigcup D_2 \bigcup ...D_n) = \sum_{i=1}^{n} p(D_i) \end{aligned}\right\} p(D) \in \mathbb{R}, p(D) \geqslant 0$$

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What is a PDF, then?

- Empirical fits to gamma functions are not PDFs
- What is a PDF?
- So for instance, this expression qualifies as a (proper) PDF:

$$p(D) = D^{\mu}\Lambda^{\mu+1}\frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$$

This is the probability of finding a drop of diameter D into the population

How do you move from PDFs to RDSDs?

- A Rain Drop Size Distribution n(D) is simply PDF times the number of drops:

$$p(D) = D^{\mu}\Lambda^{\mu+1}\frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$$

$$n(D) = N_T \cdot p(D) = N_T D^{\mu}\Lambda^{\mu+1}\frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$$

This is the number of drops of diameter D we expect to find in the population

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How this differs from other RDSDs?

- Only apparently similar to for instance Ulbrich's RDSD:

$$n(D) = N_T \cdot p(D) = N_T D^{\mu} \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$$

$$n(D)_{Ulb} = N_0 D^{\mu} e^{-\Lambda D}$$

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Why do we really need RDSDs to be probabilistic (and N_T-linked with PDFs)?

1. Mathematical consistency: robust parameter estimation requires PDFs.
2. Physical modeling: RDSD comes from a random process.
3. We want a coherent set of units.
4. To build a Z/R relationship independent on N_T
5. To analyze microphysics in terms of physically-meaningful parameters [not a and b].

Integral parameters

from

$$n(D) = N_T \cdot p(D) = N_T D^{\mu} \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$$

to

Z, R, W, Z_e, P_s, P_e, P_a …

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Z in terms of [gamma] PDFs

$$Z = k_1 \int n(D) D^6 dD = k_1 \int N_T p(D) D^6 dD =$$

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$$Z = k_1 \int n(D) D^6 dD = k_1 \int N_T p(D) D^6 dD =$$

$$= k_1 \int N_T D^\mu \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu+1)} D^6 dD =$$

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Z in terms of [gamma] PDFs

$$
\begin{aligned}
Z = k_1 \int n(D) D^6 dD = k_1 \int N_T p(D) D^6 dD = \\
= k_1 \int N_T D^{\mu} \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu+1)} D^6 dD = \\
= k_1 N_T \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} \int D^{\mu+6} e^{-\Lambda D} dD
\end{aligned}
$$

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It happens that the integral is analytically solvable

$$\int D^{\mu+6} e^{-\Lambda D} dD = \Lambda^{-(\mu+7)} \Gamma(\mu + 7)$$

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It happens that the integral is analytically solvable

$$\int D^{\mu+6} e^{-\Lambda D} dD = \Lambda^{-(\mu+7)} \Gamma(\mu+7)$$

$$Z = k_1 N_T \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} \Lambda^{-(\mu+7)} \Gamma(\mu+7) =$$

$$= k_1 N_T \Lambda^{-6} \frac{\Gamma(\mu+7)}{\Gamma(\mu+1)}$$

The same for R

Assuming this fall speed modeling $v(D) = v_1 D^{v_2}$

$$R = k_3 \cdot v_1 N_T \Lambda^{-(3+v_2)} \frac{\Gamma(\mu + v_2 + 4)}{\Gamma(\mu + 1)}$$

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But instruments have limits for min and max diameters

$$Z = k_1 N_T \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} \Lambda^{-(\mu+7)} \Gamma(\mu+7) =$$
$$= k_1 N_T \Lambda^{-6} \frac{\Gamma(\mu+7)}{\Gamma(\mu+1)}$$

$$Z = k_1 N_T \Lambda^{-6} \frac{\gamma(\mu+7, D_{max}\cdot\Lambda) - \gamma(\mu+7, D_{min}\cdot\Lambda)}{\Gamma(\mu+1)}$$

$$R = k_3 \cdot v_1 N_T \Lambda^{-(3+v_2)} \frac{\Gamma(\mu+v_2+4)}{\Gamma(\mu+1)}$$

$$R = k_3 v_1 N_T \Lambda^{-(3+v_2)} \frac{\gamma(\mu+4+v_2, D_{max}\Lambda) - \gamma(\mu+4+v_2, D_{min}\Lambda)}{\Gamma(\mu+1)}$$

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What about parameters?

N_T is clear, but what is the physical meaning of Λ and μ?

$$p(D) = D^{\mu}\Lambda^{\mu+1}\frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$$

Since p(D) is a PDF, we can apply the MOM

Method of Moments:

$$\Lambda = \frac{m}{\sigma^2}$$

$$\mu = \frac{m^2-\sigma^2}{\sigma^2}$$

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All integral parameters will now depend on m, σ^2, and N_T

$$\Lambda = \frac{m}{\sigma^2}$$

$$\mu = \frac{m^2 - \sigma^2}{\sigma^2}$$

$$Z = k_1 N_T \Lambda^{-6} \frac{\gamma(\mu + 7, D_{max} \cdot \Lambda) - \gamma(\mu + 7, D_{min} \cdot \Lambda)}{\Gamma(\mu + 1)}$$

$$R = k_3 v_1 N_T \Lambda^{-(3+v_2)} \frac{\gamma(\mu+4+v_2, D_{max}\Lambda) - \gamma(\mu+4+v_2, D_{min}\Lambda)}{\Gamma(\mu+1)}$$

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Generalized moments

$$M_{x,y} \equiv \frac{\int_0^\infty N(D) D^x dD}{\int_0^\infty N(D) D^y dD} :$$

D_m [N_T independent]

$$D_m = \Lambda^{-1} \frac{\gamma(\mu+5, D_{max}\Lambda) - \gamma(\mu+5, D_{min}\Lambda)}{\gamma(\mu+4, D_{max}\Lambda) - \gamma(\mu+4, D_{min}\Lambda)}$$

NB: $D_m = (\mu+4)/\Lambda = m + 3(\sigma^2/m)$

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W [and scaled intercept param N_w]

$$W = k_2 N_T \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} \Lambda^{-(\mu+3)} \Gamma(\mu+4) =$$

$$= k_2 N_T \Lambda^{-3} \frac{\Gamma(\mu+4)}{\Gamma(\mu+1)}$$

$$N_w \equiv \frac{4^4}{\pi \rho_w} \frac{W}{D_m^4}$$

$$N_w = \frac{4^4 k_2}{\pi \rho_w} \cdot N_T \cdot \Lambda \frac{[\gamma(\mu+4+v_2, D_{max}\Lambda) - \gamma(\mu+4+v_2, D_{min}\Lambda)]^5}{\Gamma(\mu+1)[\gamma(\mu+5, D_{max}\Lambda) - \gamma(\mu+5, D_{min}\Lambda)]^4}$$

N_w is now made explicitly dependent on m, σ^2 and N_T

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Polarimetric radar

$$Z_e = \frac{\lambda^4}{\pi^5 |K_w|^2} k_4 \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} N_T \int_{D_{min}}^{D_{max}} D^\mu e^{-\Lambda D} \sigma_b(D,\lambda) dD$$

$$k = 4.343 \cdot k_5 \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} N_T \int_{D_{min}}^{D_{max}} D^\mu e^{-\Lambda D} \sigma_e(D,\lambda) dD$$

$$P_s = k_6 \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} N_T \int_{D_{min}}^{D_{max}} D^\mu e^{-\Lambda D} \sigma_s(D,\lambda) dD$$

$$P_e = k_6 \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)} N_T \int_{D_{min}}^{D_{max}} D^\mu e^{-\Lambda D} \sigma_e(D,\lambda) dD$$

$$P_a = N_T \frac{\int_{D_{min}}^{D_{max}} D^\mu e^{-\Lambda D} \sigma_s(D,\lambda) a(D,\lambda) dD}{\int_{D_{min}}^{D_{max}} D^\mu e^{-\Lambda D} \sigma_s(D,\lambda) dD}$$

Using those equations, it is possible to retrieve N_T and build tables for m and σ^2.

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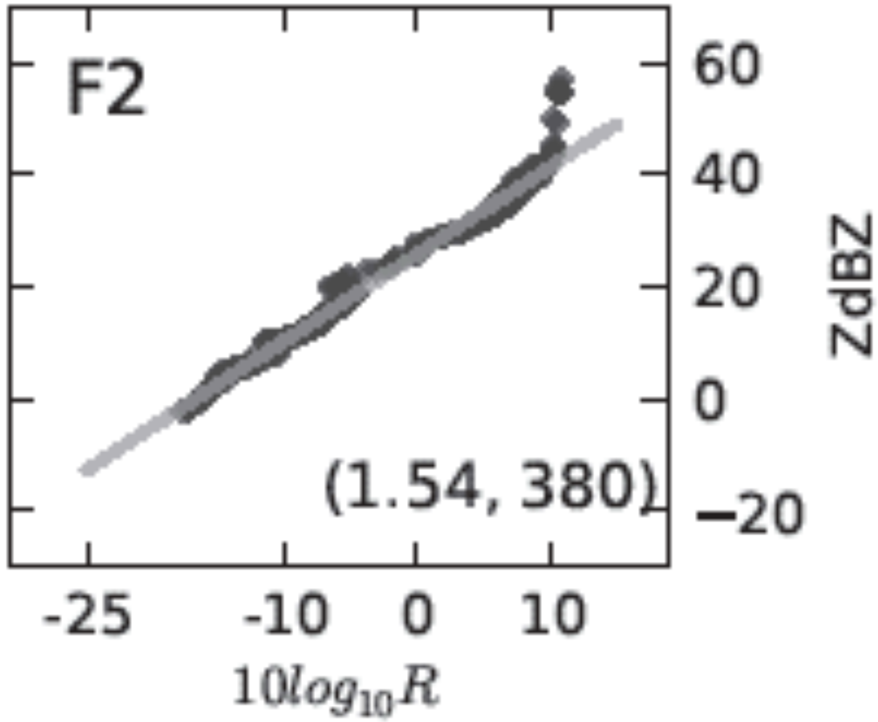
Z-R relationship

$Z=a \cdot R^b$ expressions are the result of correlating Z with R (statistical)

a and b do not have physical meaning

There is not such thing called ‘instantaneous Z-R’ [but a radar beam is pretty instantaneous]



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Natural Z-R relationship for a gamma distribution

$$\frac{Z}{R} = \frac{k_1}{k_3} \cdot v_1^{-1} \cdot \Lambda^{-(3+v_2)} \frac{\Gamma(\mu+7)}{\Gamma(\mu+4+v_2)}$$

$$R = T \cdot Z$$

$$T \equiv \frac{k_3}{k_1} \cdot v_1 \cdot \Lambda^{(3-v_2)} \frac{\gamma(\mu+4+v_2, D_{max}\Lambda) - \gamma(\mu+4+v_2, D_{min}\Lambda)}{\gamma(\mu+7, D_{max}\Lambda) - \gamma(\mu+7, D_{min}\Lambda)}$$

T doesn't depend on N_T. It only depends on m and σ^2

Gamma modeling means

You have three fundamental variables for the integral parameters:

$$N_T, \{m, \sigma^2\}$$

Z/R depends on

$$m, \sigma^2$$

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The proof is in the eating

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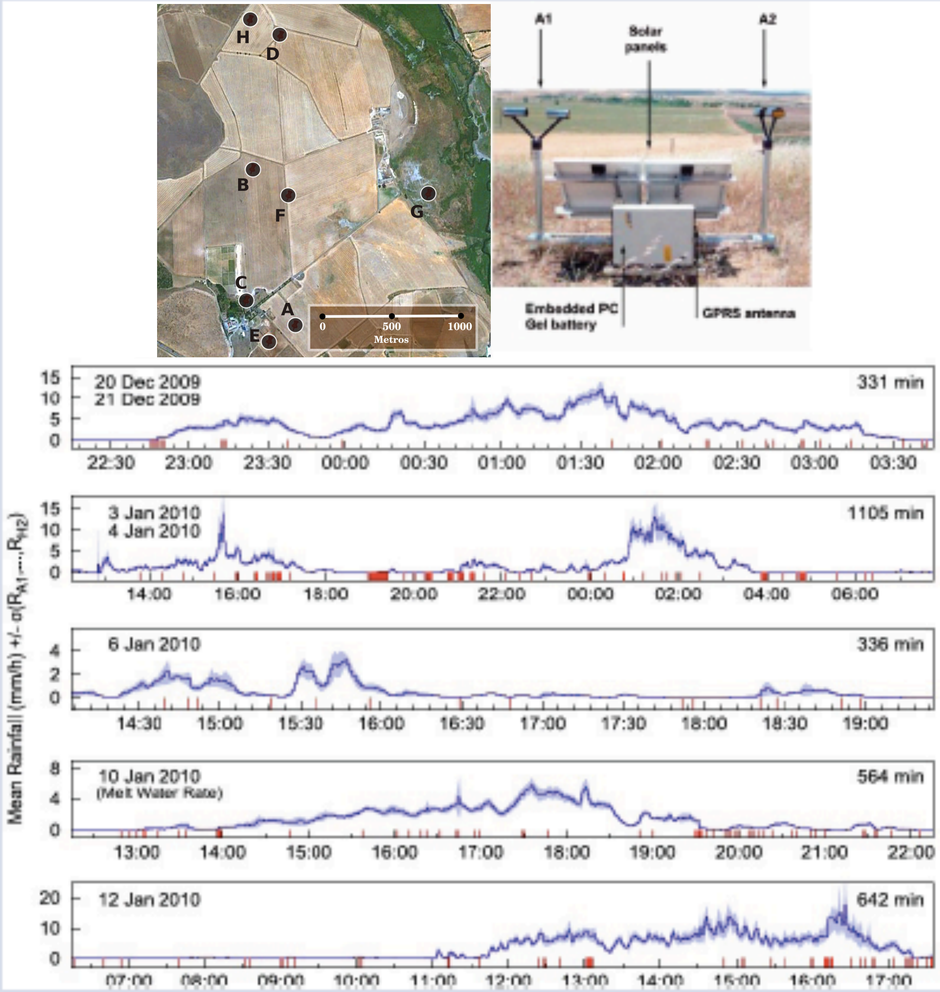
Experimental setup 1

Medium-scale variability



In 2010, we used 16 Parsivel disdrometers (in a dual setup to ensure consistency) to analyze the spatial variability of the RDSD within a DPR-size pixel.

The experiments were made in central Spain, which has a semiarid climate with moderate rain rates, and thus within Parsivels' known limitations.

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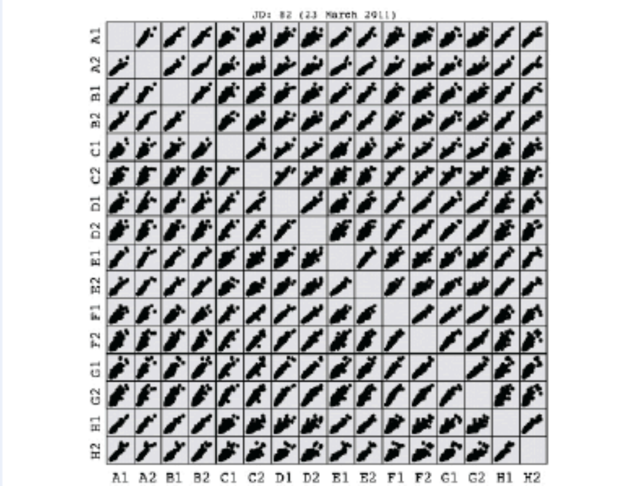
Experimental setup 2

Small-scale variability



In 2011, we located 16(+2) Parsivels to analyze the consistency of the instruments, the spatial variability of the RDSD at decimeter scale, and to cross-compare the new Parsivel² instruments.

The experiments were made in Toledo, and included a sonic anemometer.

We found that the Parsivels provided consistent estimates of the RDSD for moderate rainfall rates such as those found in Toledo.

Tapiador, F.J., Turk, J., Petersen, W., Hou, A.Y., García-Ortega, E., Machado, L.A.T, Angelis, C.F., Salio, P., Kidd, C., Huffman, G.J. and de Castro, M. 2011. Global Precipitation Measurement: Methods, Datasets and Applications. *Atmospheric Research*, accepted October 2011

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Transient setup 3

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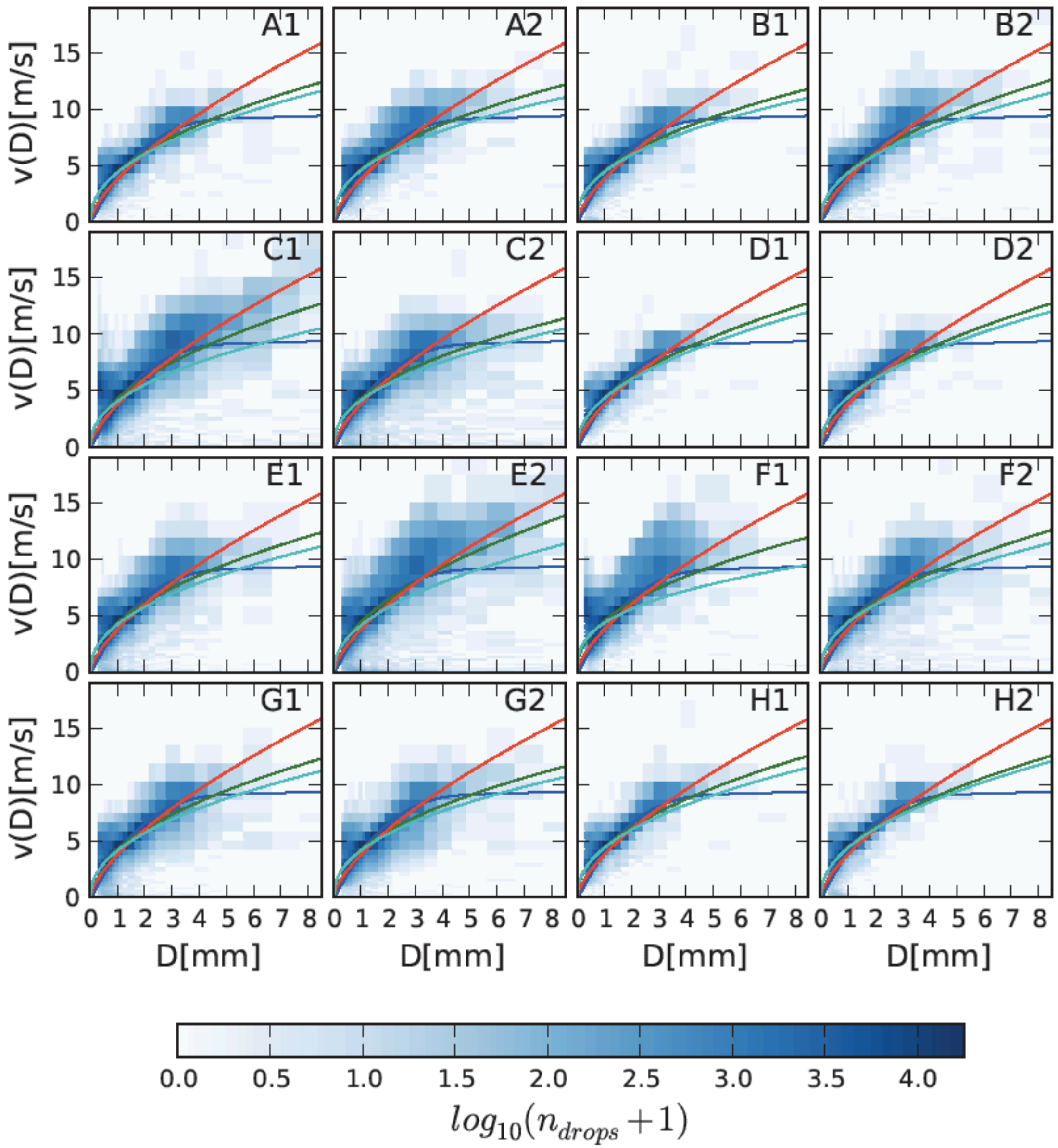
$v(D)=9.65-10.3e^{-0.6D}$

Without the filter

- Red: v(D): $3.78D^{0.67}$
- Green: linear $v(D)=\gamma D^{\delta}$
- Light Blue: non-linear $v(D)=\gamma D^{\delta}$
- Dark Blue: non-sphericity correction



Checa, R. 2012, PhD dissertation, Director: Francisco J. Tapiador

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Filtering with

$v(D)=9.65-10.3e^{-0.6D}$

With the filter

-Red: v(D): $3.78D^{0.67}$
-Green: linear $v(D)=\gamma D^{\delta}$
-Light Blue: non-linear $v(D)=\gamma D^{\delta}$
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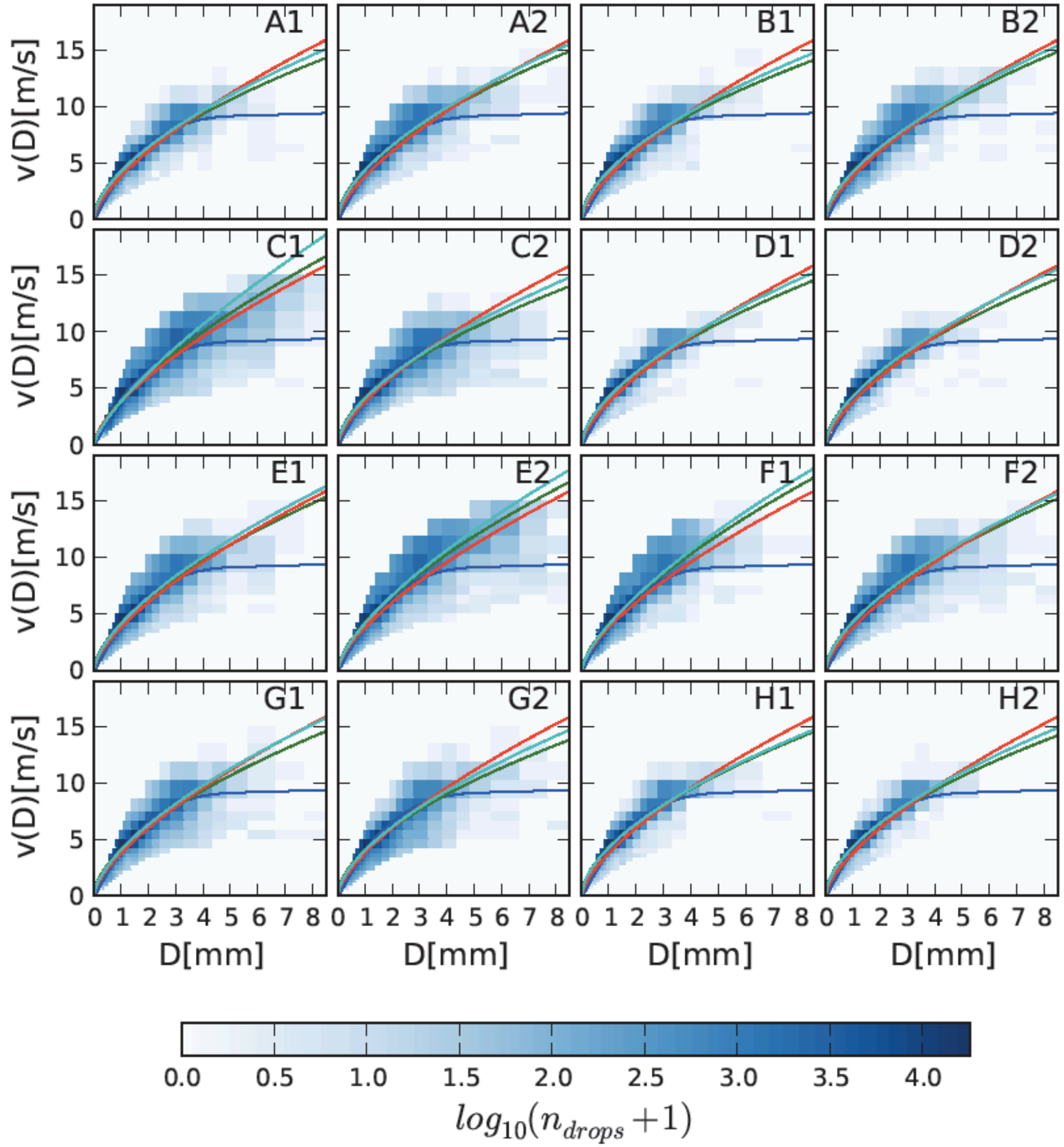

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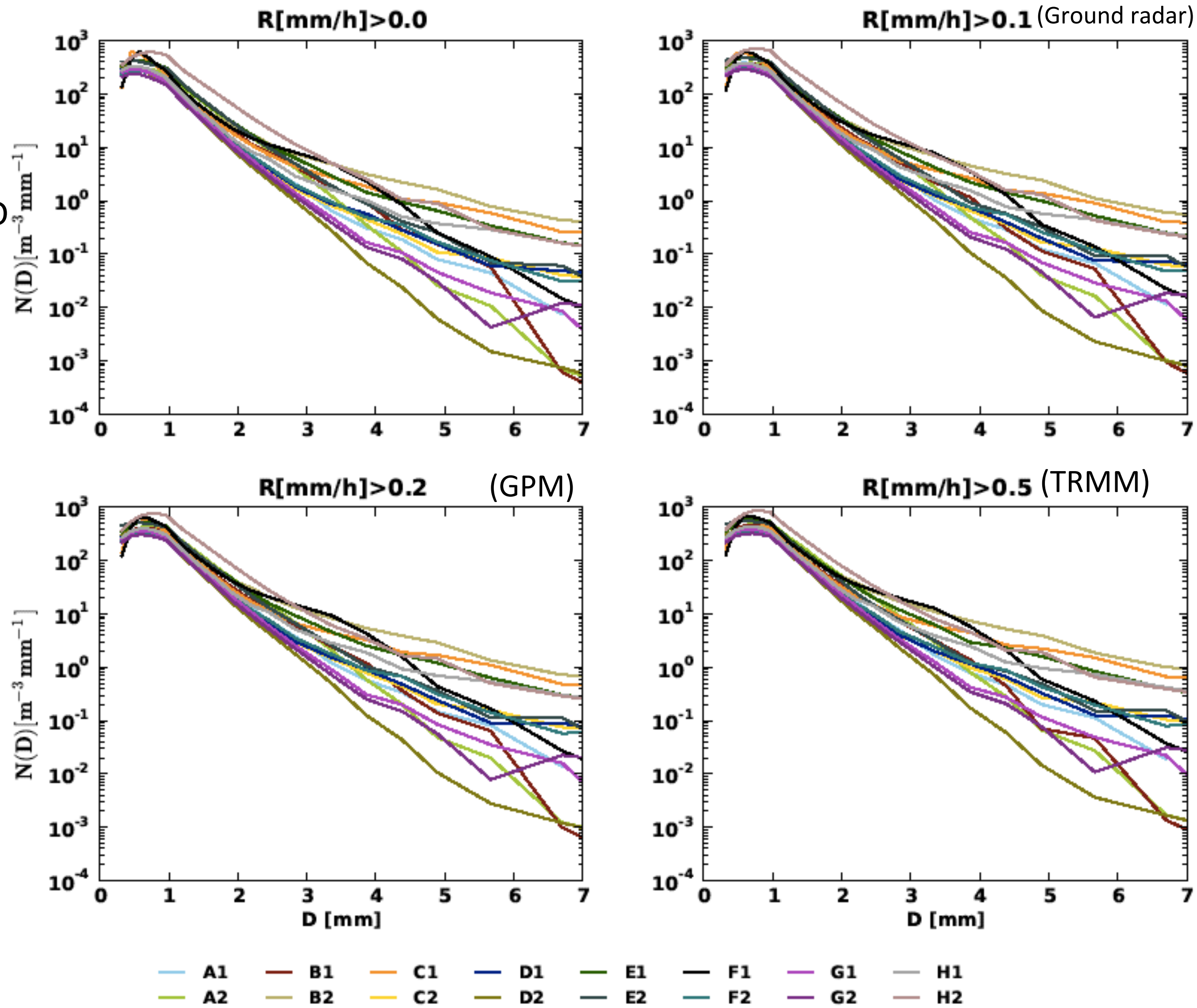
Composite DSD

Filtering with
$v(D)=9.65-10.3e^{-0.6D}$

Effects on different R thresholds

No filter



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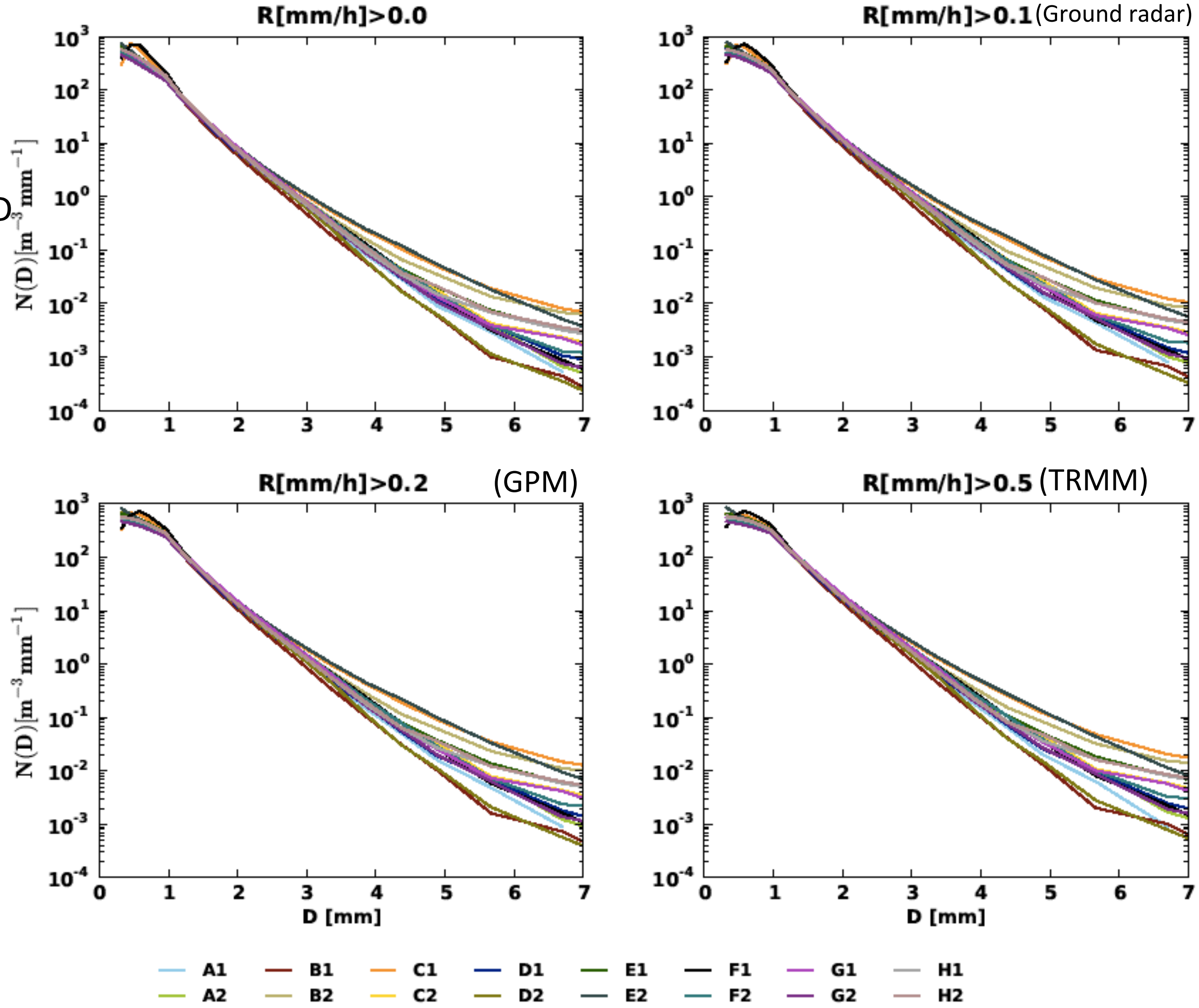
Composite DSD

Filtering with
$v(D)=9.65-10.3e^{-0.6D}$

Effects on different
R thresholds

Filter applied



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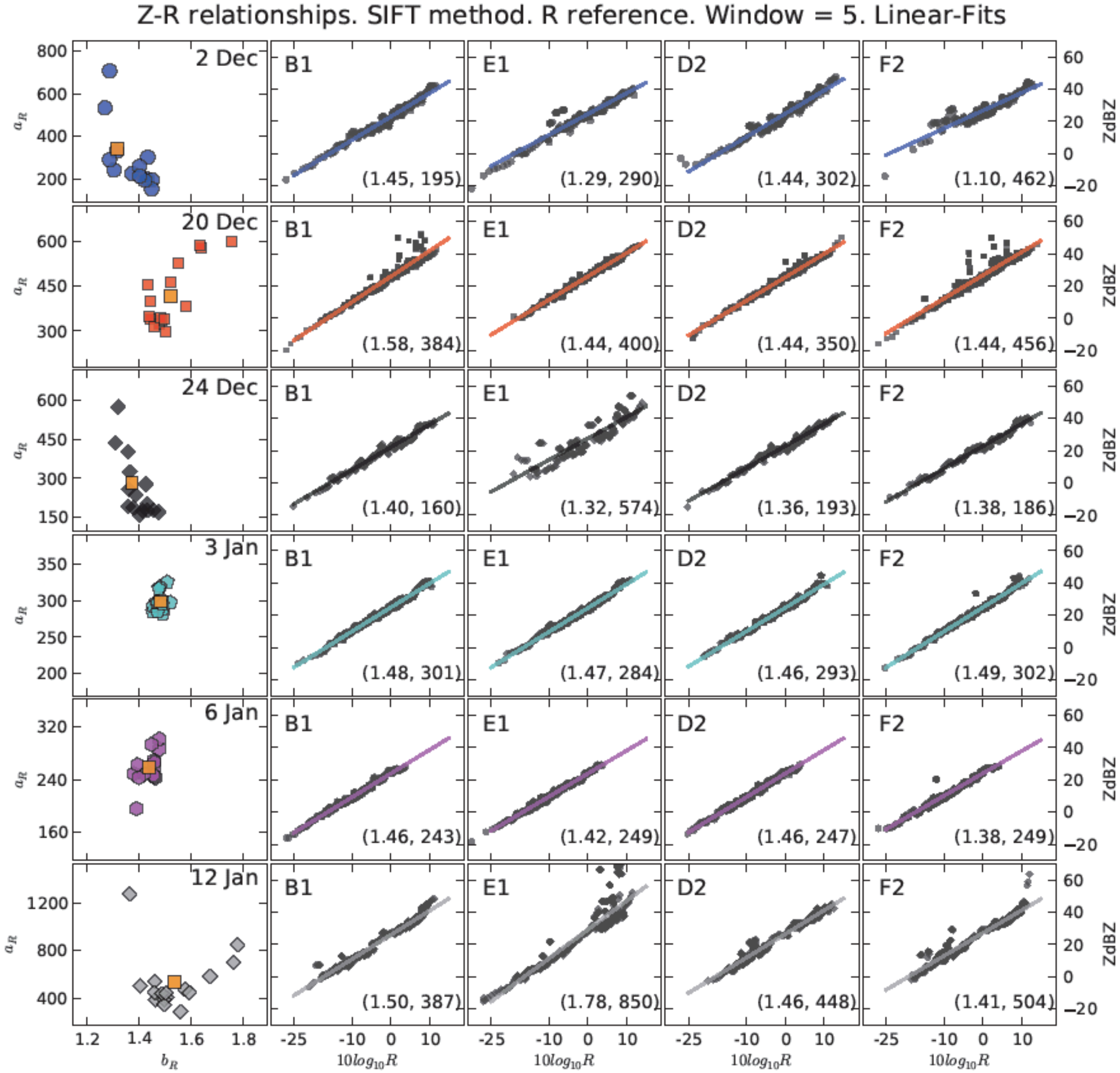

Variability of a and b

SIFT method
Window=5

[Sequential Intensity Filtering Technique, Lee and Zawadzki 2005]

Aim: filtering out stochastic variability due to poor sampling of large drops

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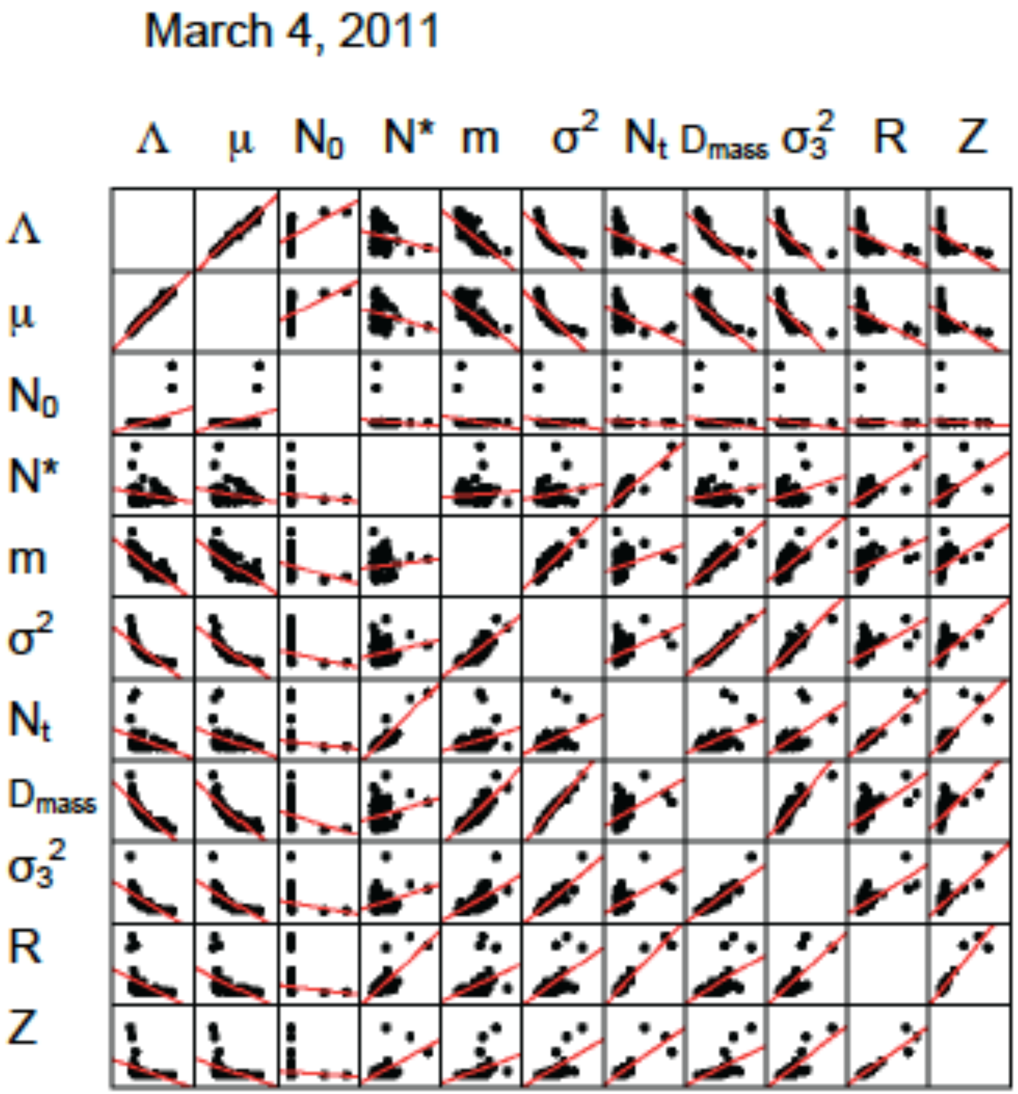
Testing the probabilistic PDF

Cross-correlations for
one disdrometer,
four episodes,
5 min accumulation



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a) We measure *m*, σ^2, *R* and *Z* with a disdrometer

b$_1$) We calculate T=R/Z from measured R and Z [call it T empirical]

b$_2$) We also use the analytical formulae to derive T [call it T analytical] using two values: *m* and σ^2

$$R = T \cdot Z$$

$$T \equiv \frac{k_3}{k_1} \cdot v_1 \cdot \Lambda^{(3-v_2)} \frac{\gamma(\mu+4+v_2, D_{max}\Lambda) - \gamma(\mu+4+v_2, D_{min}\Lambda)}{\gamma(\mu+7, D_{max}\Lambda) - \gamma(\mu+7, D_{min}\Lambda)}$$

$$\Lambda = \frac{m}{\sigma^2}$$
$$\mu = \frac{m^2 - \sigma^2}{\sigma^2}$$

Parameters
$k_1 = 1.0$ for $[Z] = mm^6 \cdot m^{-3}$
$k_2 = \frac{\pi}{6000}$ for $[W] = mm^3 \cdot m^{-3}$
$k_3 = \frac{6\pi}{10^4}$ for $[R] = mm \cdot h^{-1}$
$v_1 = 3.78,\ v_2 = 0.67$

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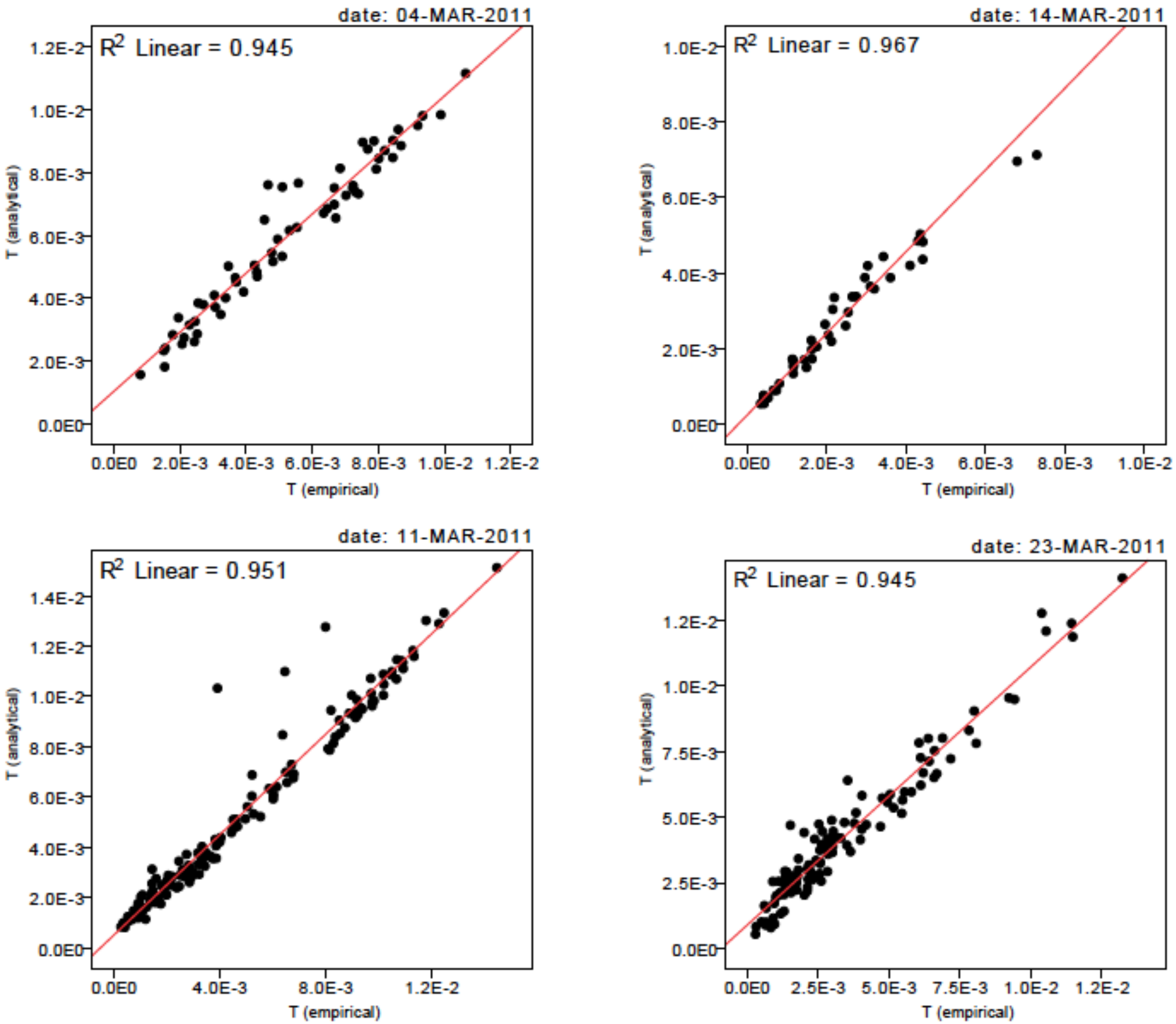
R/Z (empirical)

vs.

R/Z (analytical)



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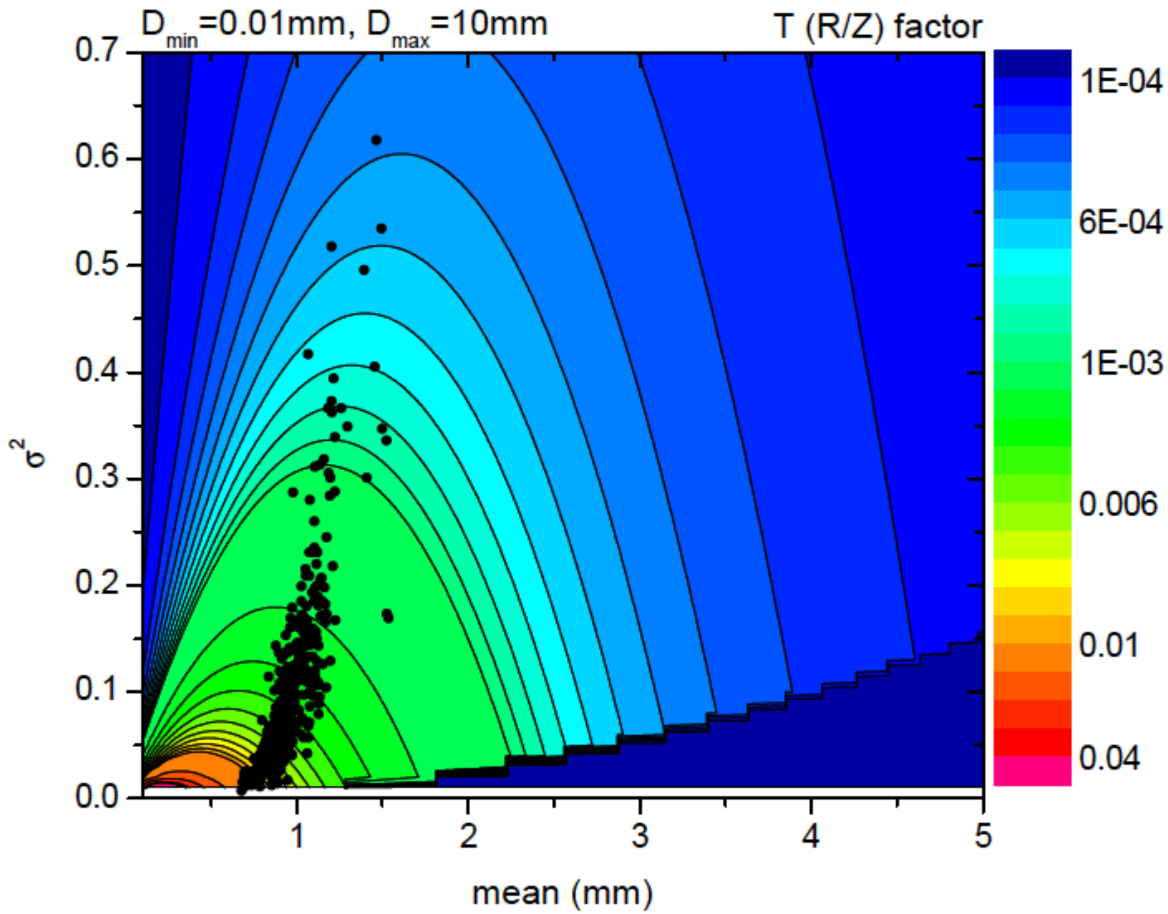
Real rainfall in *m* and σ^2 space

Measured rain (dots) is restricted to a narrow band:



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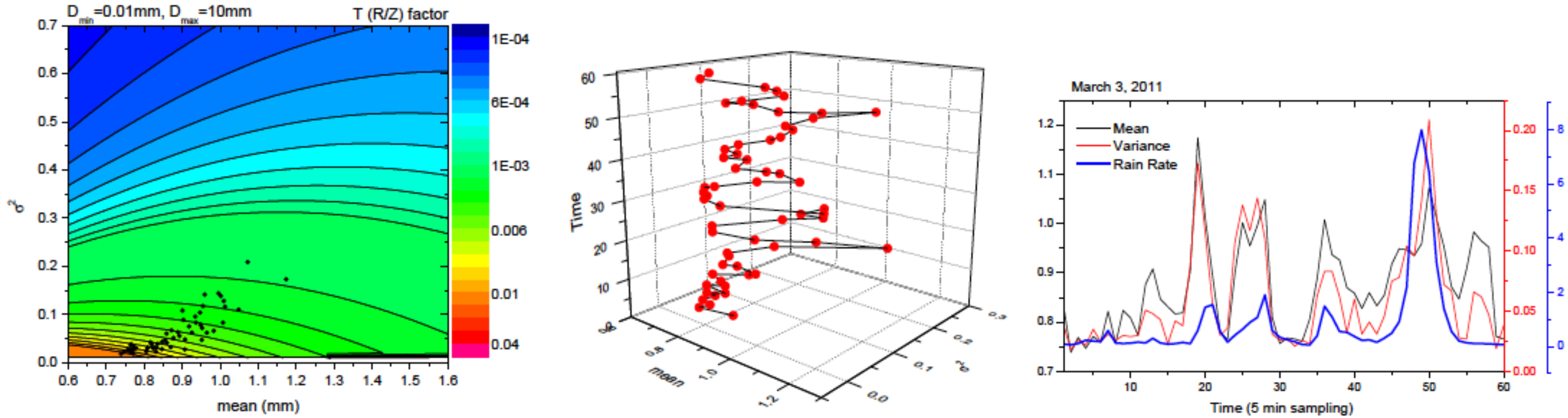
Time-evolution of *m* and σ^2 (and thus time-evolution of T=R/Z)



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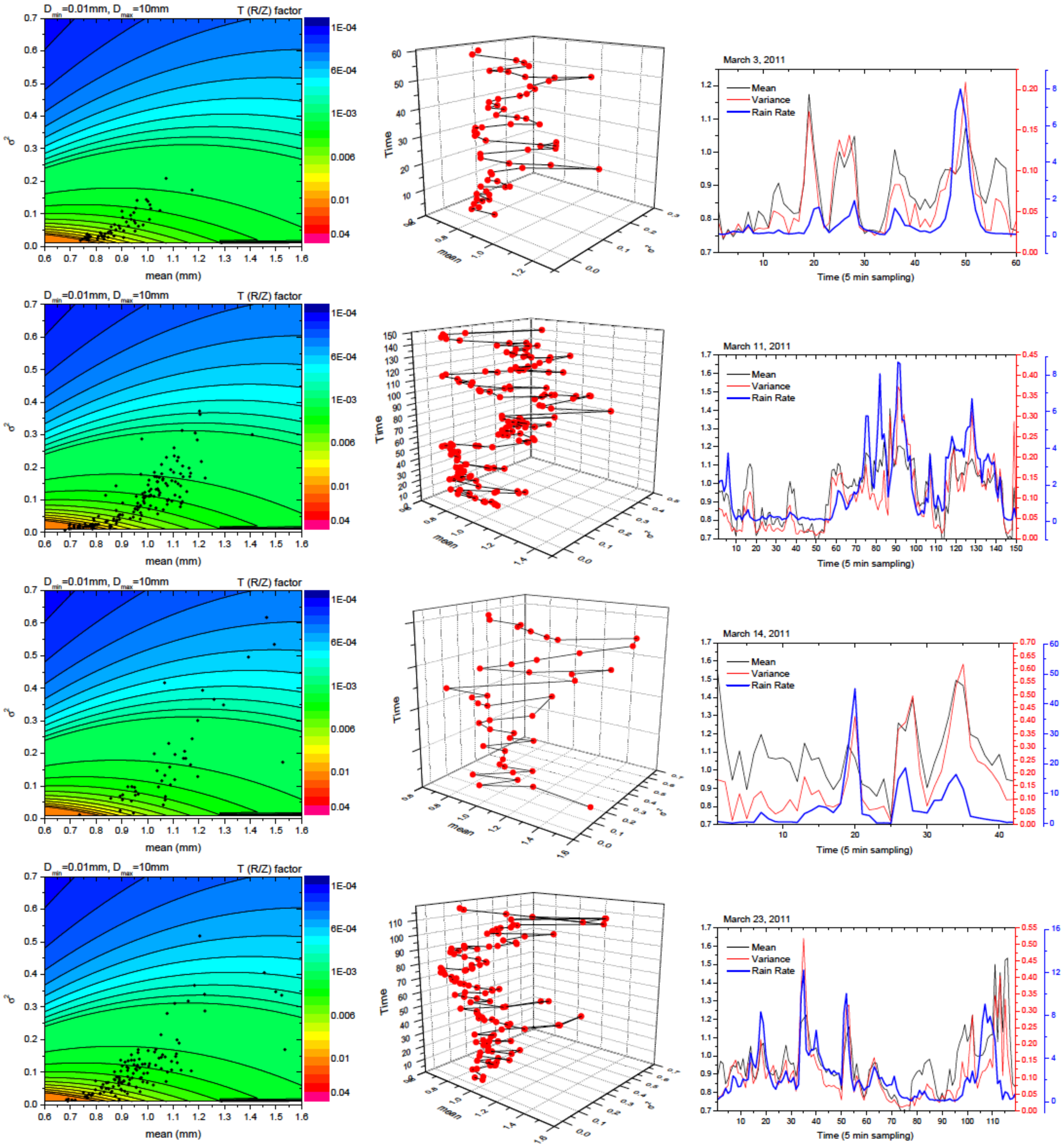
Evolution for 4 episodes



2013 NASA Precipitation
Measurement Missions (PMM)
Science Team Meeting

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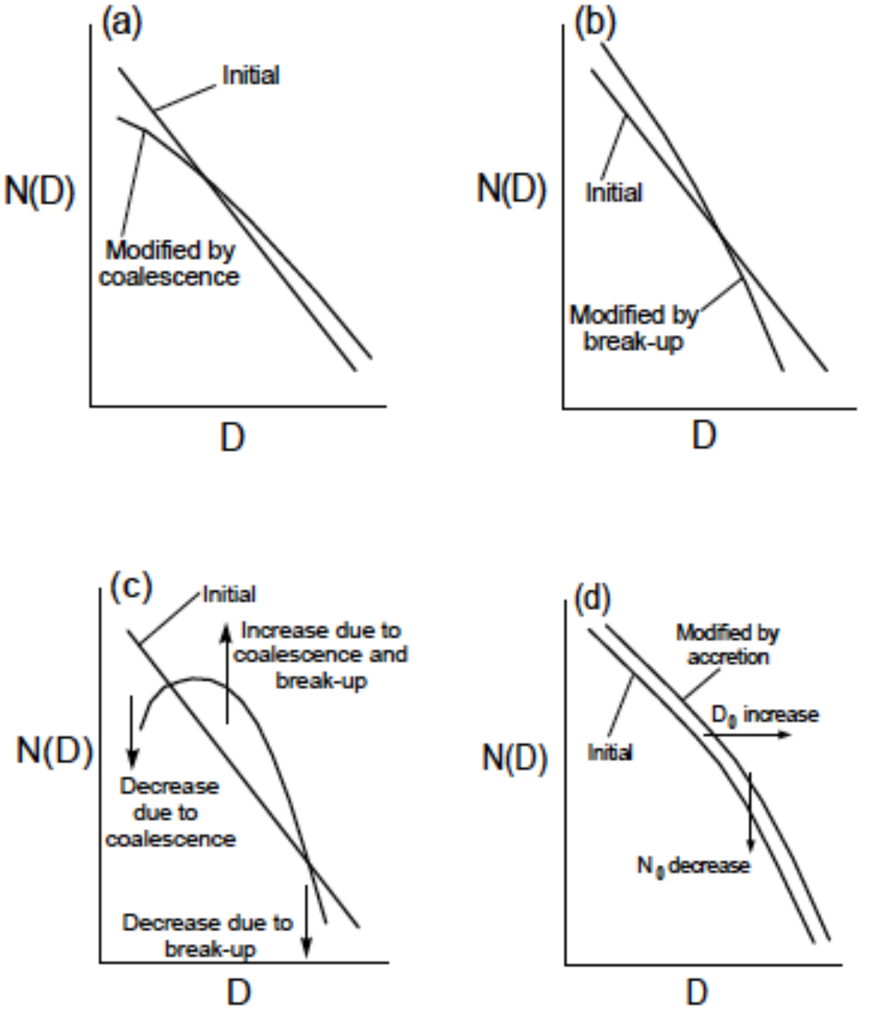

Fig. 3 (a)-(d): Schematic diagrams illustrating the effects on the raindrop size distribution of (a) raindrop coalescence, (b) raindrop break-up, (c) coalescence and break-up acting simultaneousl and (d) accretion of cloud droplets.

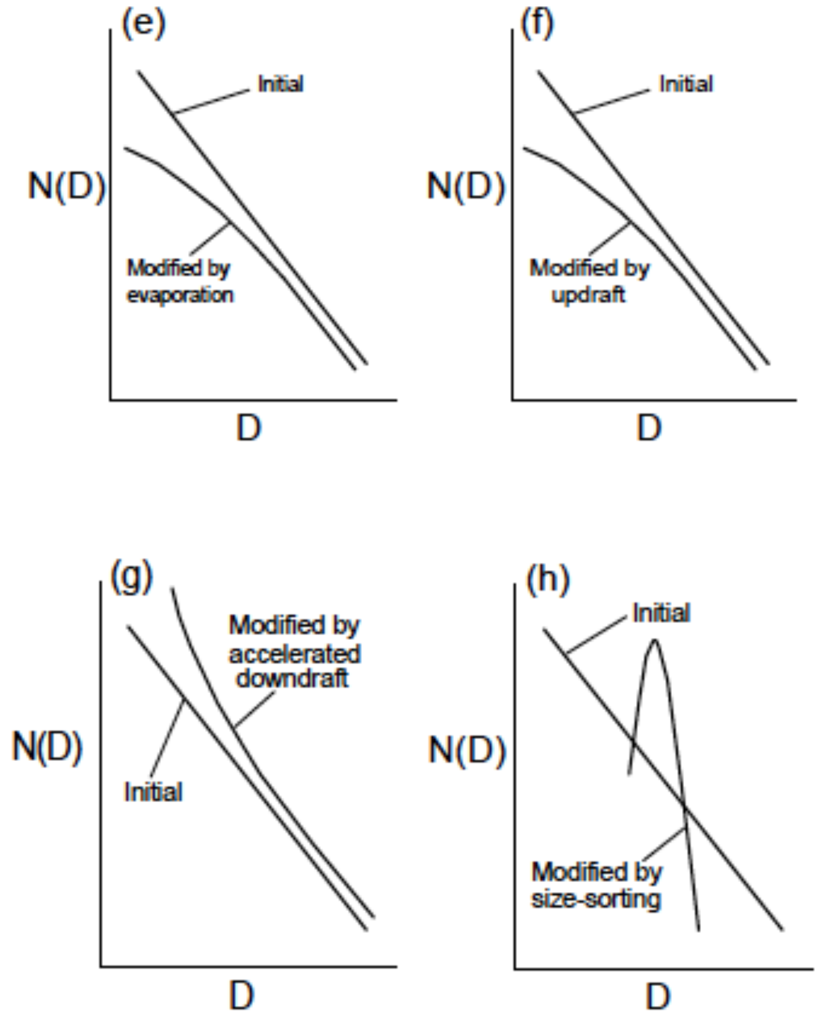

Fig. 3(e)-(g): Schematic diagrams illustrating the effects on the raindrop size distribution of (e) evaporation, (f) an updraft, (g) an accelerated downdraft, and (h) size-sorting.

Rosenfeld, Daniel, Carlton W. Ulbrich, 2003: Cloud Microphysical Properties, Processes, and Rainfall Estimation Opportunities. *Meteorological Monographs*, **30**, 237–237.

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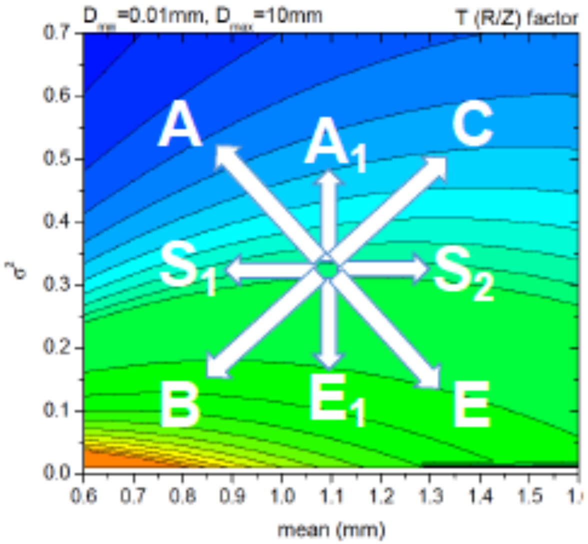
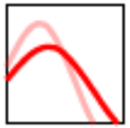
Microphysics



A	m	↓	small drops	↑	Accretion
	σ^2	↑	large drops	↑	
B	m	↓	small drops	↑	Break-up
	σ^2	↓	large drops	↓	
C	m	↑	small drops	↓	Coalescence
	σ^2	↑	large drops	↑	
E	m	↑	small drops	↓	Evaporation
	σ^2	↓	large drops	↓	
			large drops	↑	Coalescence
S_1	m	↓	small drops	↓	Size sorting
	σ^2	=	large drops	↓	
S_2	m	↑	small drops	↑	Size sorting
	σ^2	=	large drops	↑	
A_1	m	=	small drops	↑	Accretion
	σ^2	↑	large drops	↑	
E_1	m	=	small drops	↓	Evaporation
	σ^2	↓	large drops	↓	



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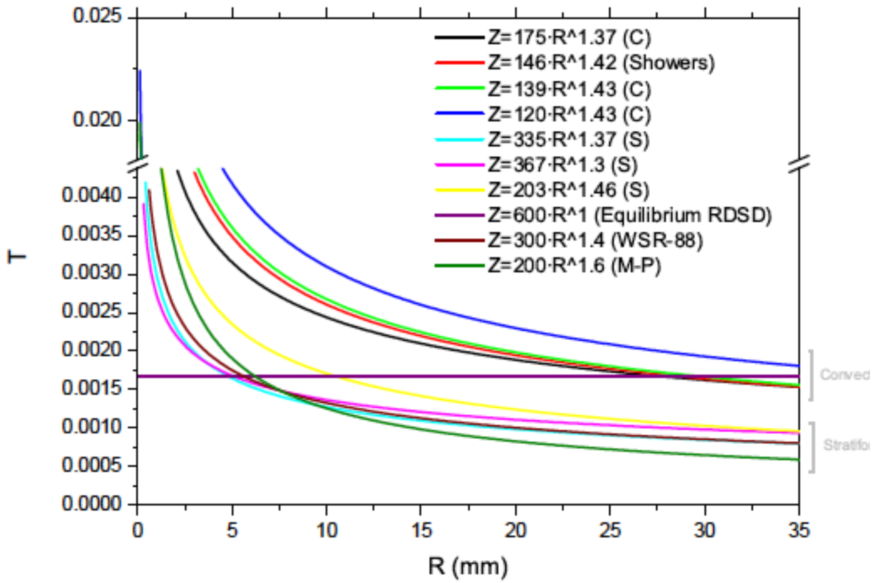
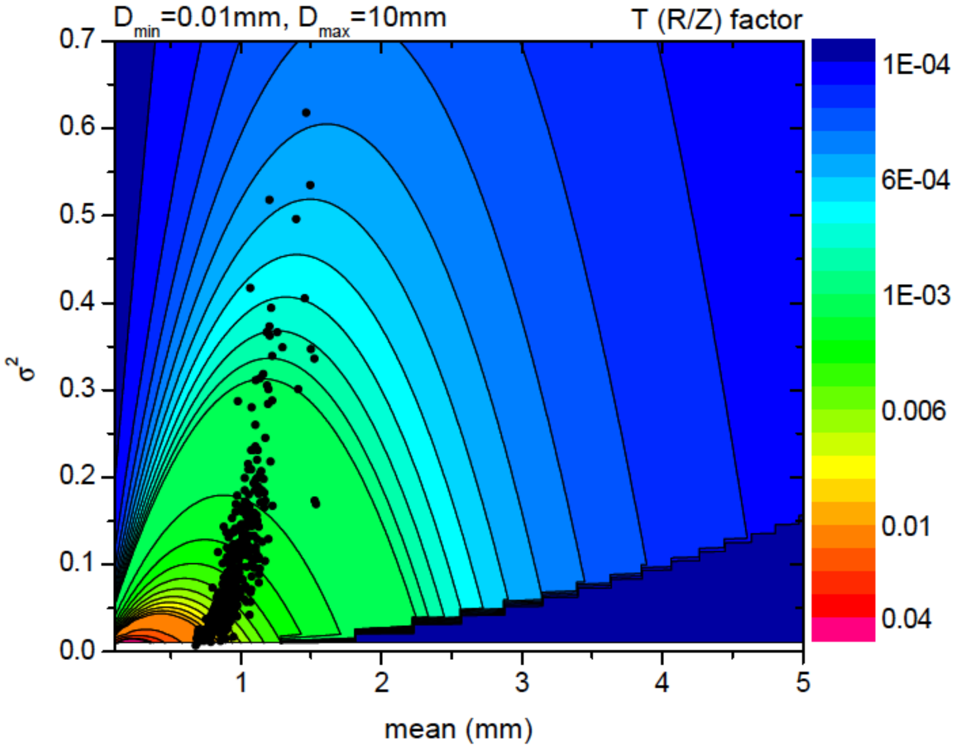


C/S for several standard Z-R relationships



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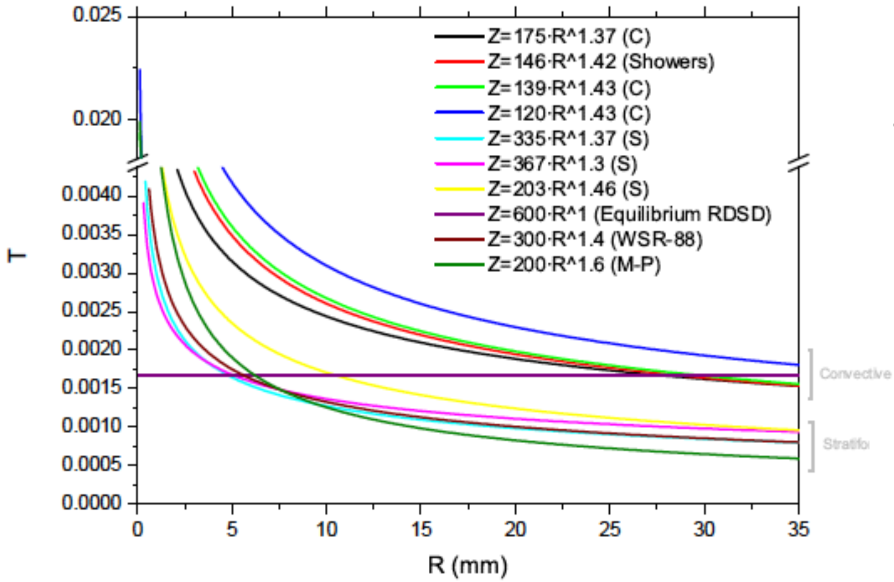
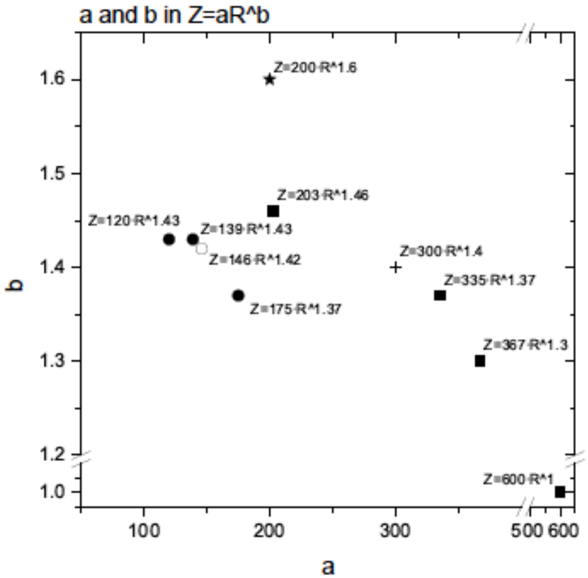
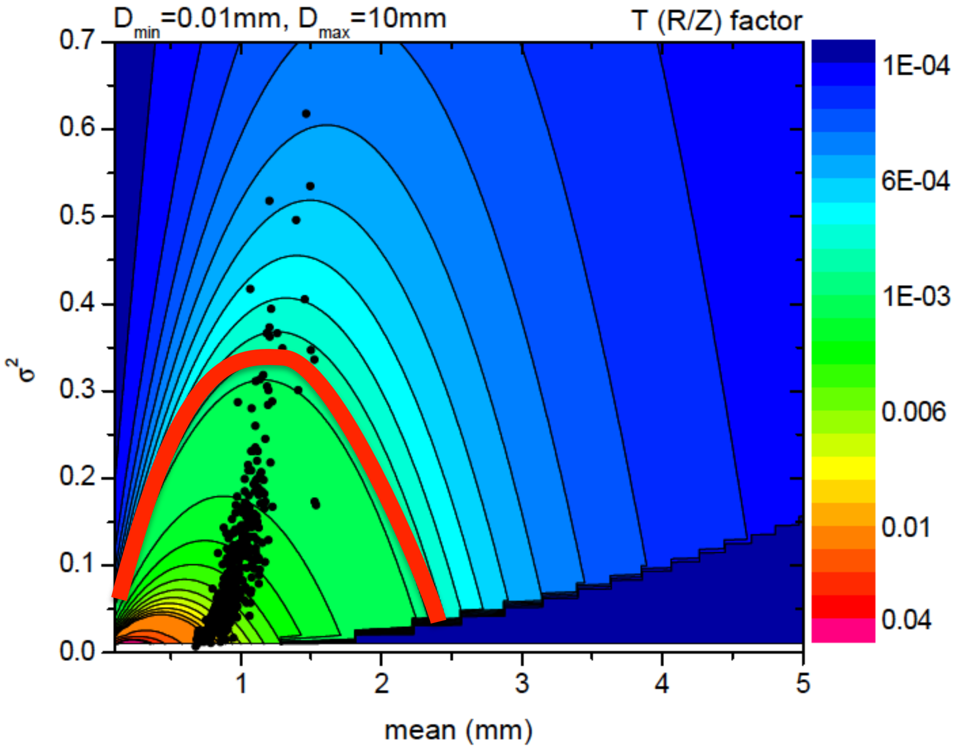


Microphysically, C/S transantes into having or not a large mean diameter

(and thus large variance)





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But the nicest thing about the probabilistic approach is that

you don't have to limit yourself to the gamma distribution

or to two parameters

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In probability theory, there is a general PDF form:

$$p(D) = exp(-\sum_{i=0}^{n} \lambda_i g_i(D))$$

$$p(D) = exp(-\sum_{i=0}^{n} \lambda_i g_i(D))$$

comes from

Maximize this function: $S = -k \int p(D) log(p(D)) dD$

Subject to: $E\{g_i(D)\} = \int p(D) g_i(D) dD \quad i = 1, ..., n$

Thus for instance

Maximize this function: $S = -k \int p(D) log(p(D)) dD$

Subject to:
$$\begin{cases} \int p(D) D dD = E\{D\} \\ \int p(D) log(D) dD = E\{log(D)\} \end{cases}$$

Yields: $$p(D) = D^{\mu} \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$$

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Therein, the gamma distribution is the PDF arising if you assume that:

1. Drop diameters are monotonically growing values, i.e.

$$\int p(D) D dD = E\{D\}$$

2. Drop diameters follow a scaling law, i.e.

$$\int p(D) log(D) dD = E\{log(D)\}$$

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In terms of stochastic processes…

If we hypothesize that rain is a stochastic, Poisson process, theory dictates that the waiting time for a drop of a given diameter to be found follows a Gamma distribution

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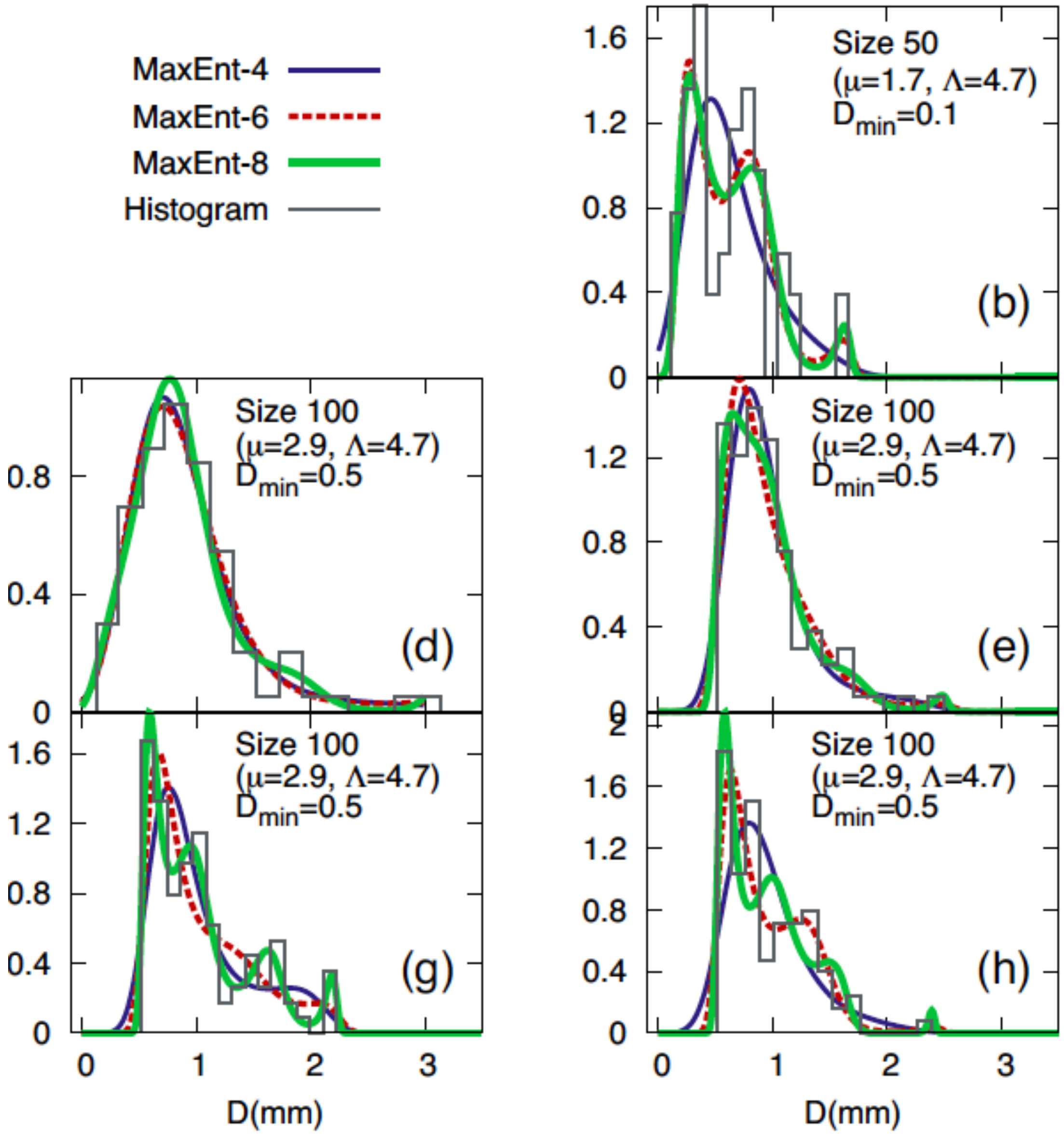
Some other choices



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Summary

Modeling the RDSD with 3 fundamental variables:

$$N_T, \{m, \sigma^2\}$$

Z/R depends on:

$$m, \sigma^2$$

Microphysics can be transparently described in terms of:

$$m, \sigma^2$$

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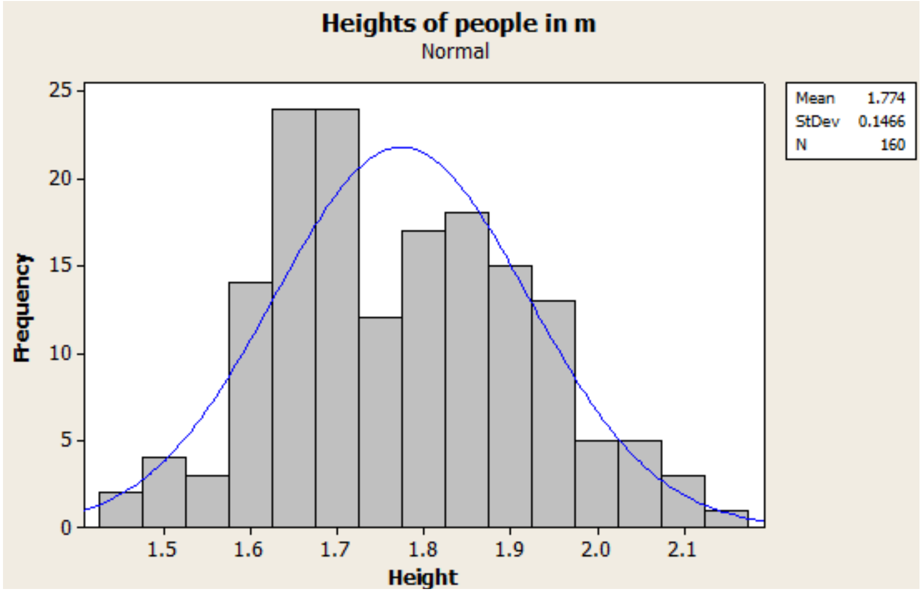
Thanks

Rain Drop Size Distribution
$n(D) = N_T D^{\mu} \Lambda^{\mu+1} \frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$
Independent variables N_T, m, σ^2
$\Lambda = \frac{m}{\sigma^2}$ $\mu = \frac{m^2-\sigma^2}{\sigma^2}$
Parameters
$k_1 = 1.0$ for $[Z] = mm^6 \cdot m^{-3}$ $k_2 = \frac{\pi}{6000}$ for $[W] = mm^3 \cdot m^{-3}$ $k_3 = \frac{6\pi}{10^4}$ for $[R] = mm \cdot h^{-1}$ D_{min} diameter of the smallest measurable drop, in mm D_{max} diameter of the largest measurable drop, in mm $v_1 = 3.78$, $v_2 = 0.67$ (Atlas and Ulbrich 2000)
Main moments (depend on N_t, m and σ^2)
$Z = k_1 N_T \Lambda^{-6} \frac{\gamma(\mu+7, D_{max}\Lambda)-\gamma(\mu+7, D_{min}\Lambda)}{\Gamma(\mu+1)}$ $W = k_2 N_T \Lambda^{-3} \frac{\gamma(\mu+4, D_{max}\Lambda)-\gamma(\mu+4, D_{min}\Lambda)}{\Gamma(\mu+1)}$ $R = k_3 v_1 N_T \Lambda^{-(3+v_2)} \frac{\gamma(\mu+4+v_2, D_{max}\Lambda)-\gamma(\mu+4+v_2, D_{min}\Lambda)}{\Gamma(\mu+1)}$ $D_m = \Lambda^{-1} \frac{\gamma(\mu+5, D_{max}\Lambda)-\gamma(\mu+5, D_{min}\Lambda)}{\gamma(\mu+4, D_{max}\Lambda)-\gamma(\mu+4, D_{min}\Lambda)}$, does not depend on N_T
Z-R relationship (depend on m and σ^2)
$R = T \cdot Z$ $T \equiv \frac{k_3}{k_1} \cdot v_1 \cdot \Lambda^{(3-v_2)} \frac{\gamma(\mu+4+v_2, D_{max}\Lambda)-\gamma(\mu+4+v_2, D_{min}\Lambda)}{\gamma(\mu+7, D_{max}\Lambda)-\gamma(\mu+7, D_{min}\Lambda)}$
Scaled intercept parameter N_w (depends on N_t, m and σ^2)
$N_w = \frac{4^4 k_2}{\pi \rho_w} \cdot N_T \cdot \Lambda \frac{[\gamma(\mu+4+v_2, D_{max}\Lambda)-\gamma(\mu+4+v_2, D_{min}\Lambda)]^5}{\Gamma(\mu+1)[\gamma(\mu+5, D_{max}\Lambda)-\gamma(\mu+5, D_{min}\Lambda)]^4}$

Tapiador, FJ, Haddad, ZS and Turk, FJ, 2013. *A Probabilistic View on the RDSD* (submitted to J. Hydromet.)

Probability vs. Statistics

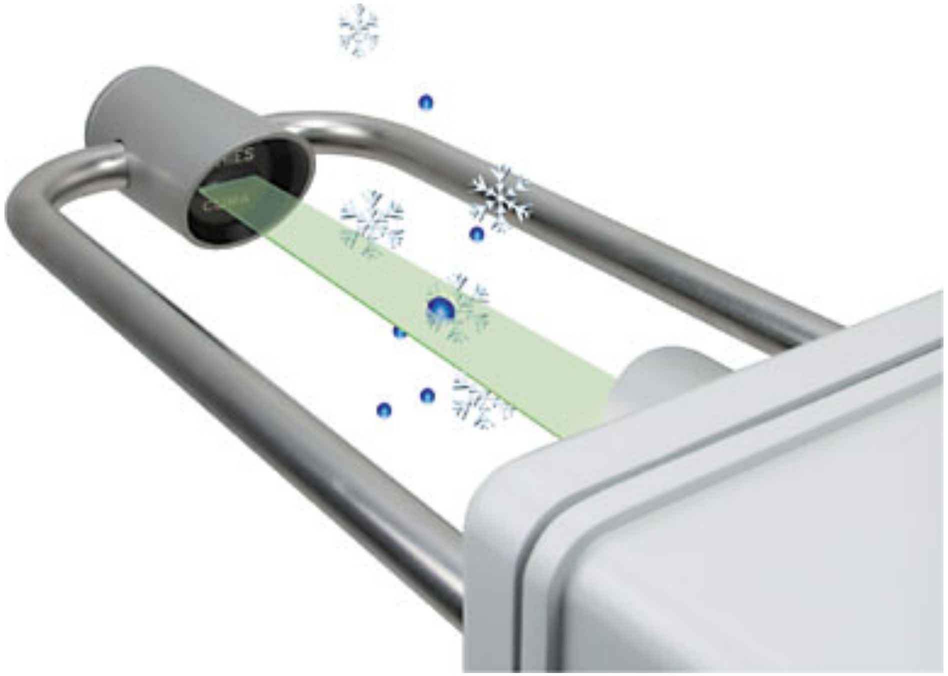
We don't need to carry out any measurement to know that the chances of drawing a King are 4/52=1/13

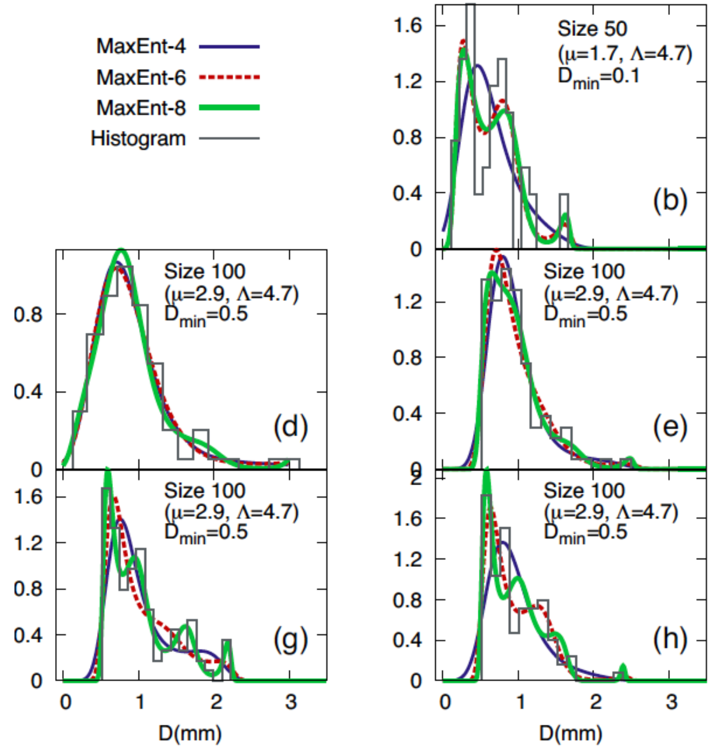


We cannot know the average size of people without performing measurements on a representative sample of the population.

[a large number of draws confirms this]

Probability vs. Statistics





We **hypothesize** that rain is a stochastic, Poisson process. Then, by crushing numbers we get that the waiting time for a drop of a given diameter to be found follows a Gamma distribution

We **measure** drops and **then** we fit the histogram to a curve. We chose a Gamma distribution because it minimizes the RMSE.

But that creates fitting artifacts