Vertical Structure of DSD Parameters Retrieved from Profilers

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Collaboration with the NASA PMM DSD Working Group:

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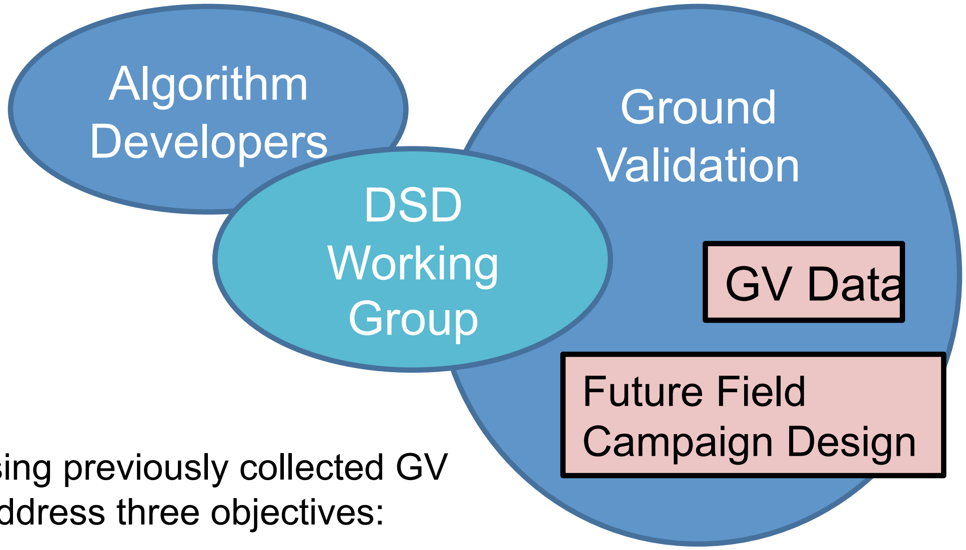
Outline

- Charter and Objectives of DSD Working Group
- Review the Darwin profiler data set from TWP-ICE
- Review some DSD Parameters (W, Nw, Dm, σ_m)
- Nw vs. Dm correlations
- σ_m vs. Dm correlations
- μ - Dm relationship - Better than a constant μ?
- Concluding Remarks
- Next Steps

DSD Working Group: Bridging Algorithms and GV

General Objective: Investigate the correlation between DSD parameters using GV data sets that support, or guide, the assumptions used in satellite retrieval algorithms.
Rational: Understanding the correlations between DSD parameters will reduce the degrees of freedom in the algorithms that must retrieve rain rates when constrained by a finite number of satellite observations.



With guidance from Algorithm Developers, we are using previously collected GV data (point, columnar, and spatial GV data sets) to address three objectives:

Objective A. Develop physically based relationships (or correlations) between DSD parameters to reduce the spread of retrieved rain rates with as few DSD parameters as possible.
Example Question:
• If the DSD is parameterized by two correlated DSD parameters (Nw and Dm), what is the spread in R given Dm and Nw(Dm)?
• How much of this variability be explained by adding a third DSD parameter (mu)?

Objective B. Investigate the degrees of freedom needed to describe the vertical structure of Nw, Dm, and mu.
Example Question:
• Is it sufficient to describe DSD parameters at the top or bottom of the column?
• How much vertical variation is observed?
•Can normalized DSD parameters help reduce the vertical variation?

Objective C. Investigate relationships between observed snow particles and bulk quantities.
Example Question:
• What are observed maximum to mean diameter ratios for different snow regimes?

Moving Forward: If GV data cannot address these objectives, then new GV data will be collected in future GV field campaigns .

Profiler Data Set

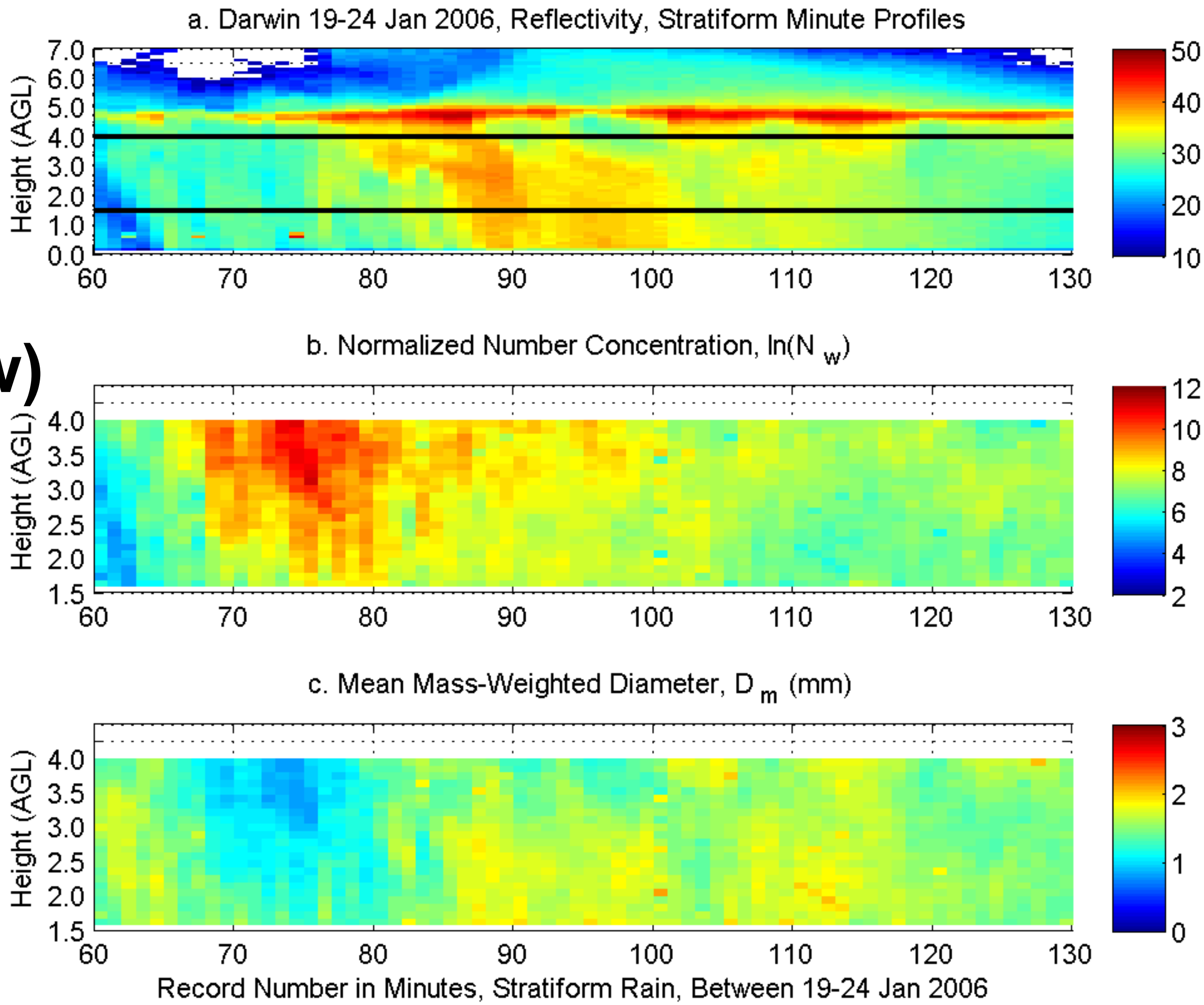
- Darwin, 50 MHz / 920 MHz profilers during TWP-ICE
- 19-24 January 2006
- Stratiform rain
  - Vertical air motion magnitude less than 1.5 m s^{-1}
  - 896 1-minute profiles
- 100 m vertical resolution
- 1.5 to 4 km (24 range gates)
- Retrieval method:
  - Vertical air motion estimated by 50 MHz profiler
  - Shift and deconvolve the 920 MHz profiler spectra
  - No fitting is performed (no assumed gamma distribution)
  - Output is a discrete N(D) at each range gate
  - **Disdrometer-like output: Number of drops in each diameter**

70 Minutes of Profiler Z, Nw, and Dm

Z

ln(Nw)

Dm



Gamma Shaped Raindrop Size Distribution: N_w, D_m, μ

$$N(D) = N_0 D^u \exp\left(-(4+\mu)\frac{D}{D_m}\right)$$

$$N(D) = N_w f(\mu)\left(\frac{D}{D_m}\right)^{\mu} \exp\left(-(4+\mu)\frac{D}{D_m}\right)$$

$$N_w = \frac{4^4 \cdot 10^3}{\pi \rho_w}\left(\frac{W}{D_m^4}\right)$$

$$f(\mu) = \frac{6}{4^4}\frac{(4+\mu)^{\mu+4}}{\Gamma(\mu+4)}$$

- Three **correlated** DSD parameters N_w, D_m, μ
- Can GV data be used to describe the correlations?

Mass Spectrum Parameters, W, D_m, σ_m

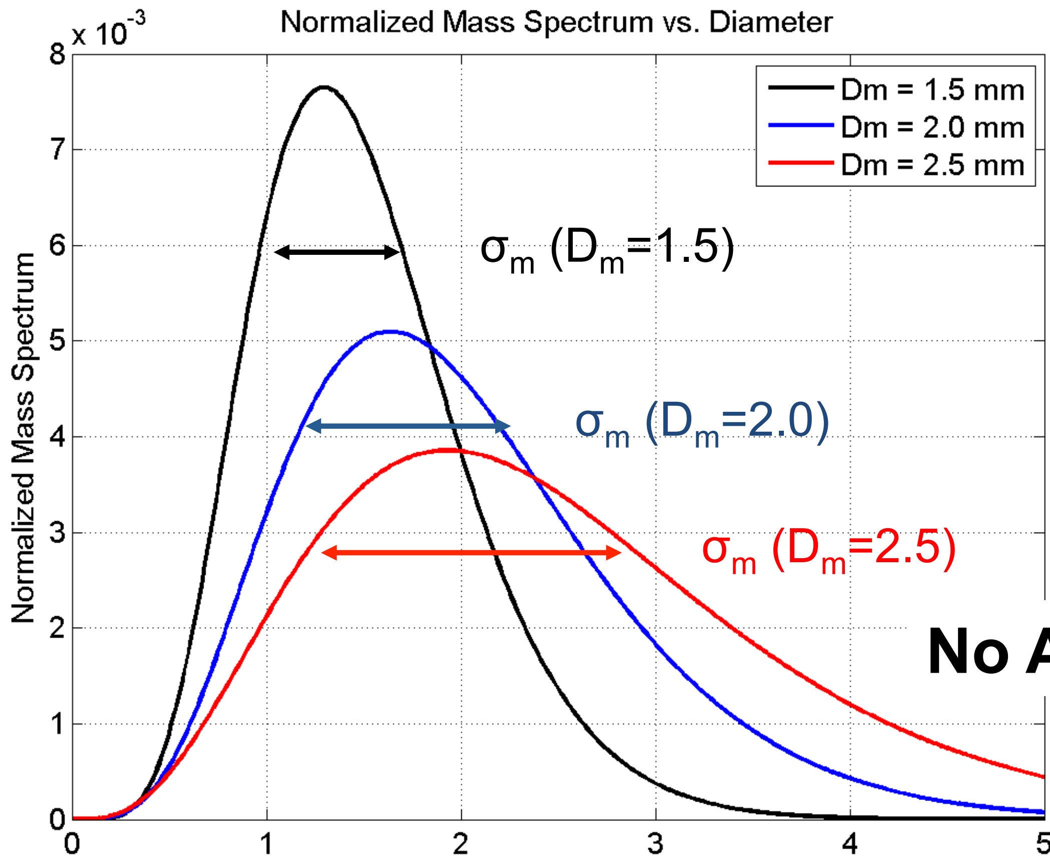
- Mass spectrum: $$M(D) = \frac{\pi}{6 \cdot 10^3} \rho_w N(D) D^3$$
- Liquid Water Content:

:

$$W = \sum M(D) dD = \frac{\pi}{6 \cdot 10^3} \rho_w \sum N(D) D^3 dD$$

- Mean mass-weighted diameter: $$D_m = \frac{\sum N(D) D^4 dD}{\sum N(D) D^3 dD} = \frac{M_4}{M_3}$$
- Mass spectrum variance: $$\sigma_m^2 = \frac{\sum (D - D_m)^2 N(D) D^3 dD}{\sum N(D) D^3 dD}$$
- Mass spectrum standard deviation, σ_m

Mass Spectrum Parameters, W, D_m, σ_m



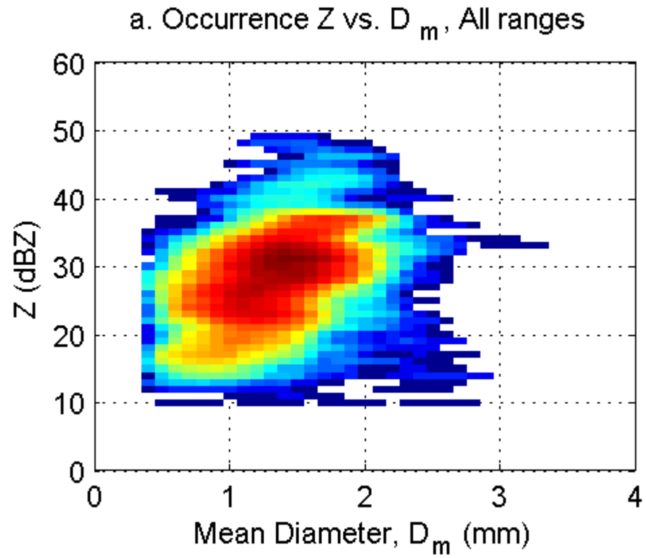
$$W = \sum N(D) D^3 dD$$

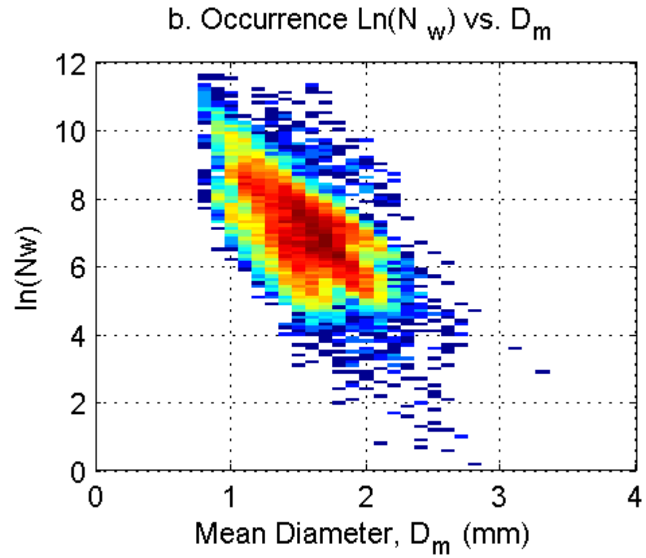
$$N_w = \frac{4^4 \cdot 10^3}{\pi \rho_w} \left(\frac{W}{D_m^4} \right)$$

No Assumed DSD Shape

Frequency of Occurrences

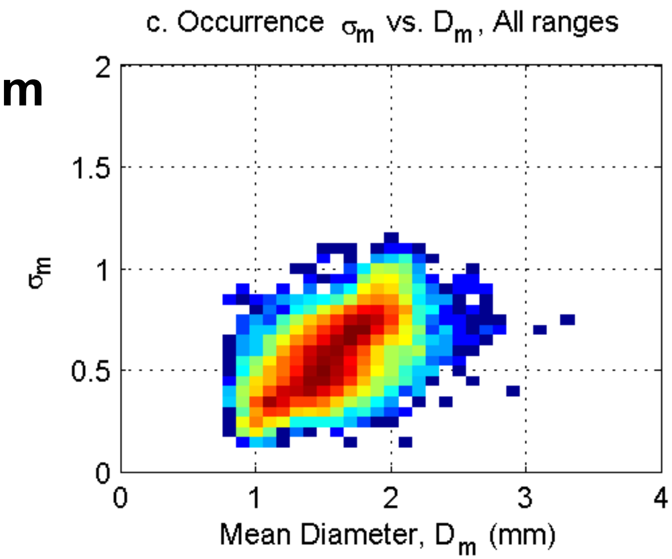
Z





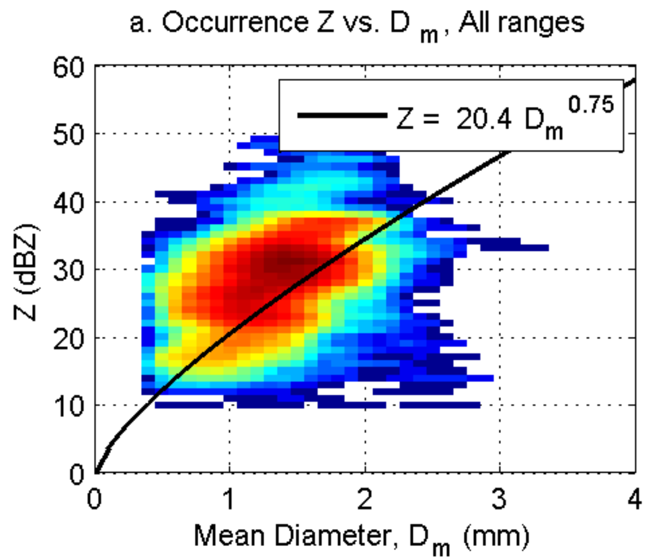
ln(Nw)

σ_m

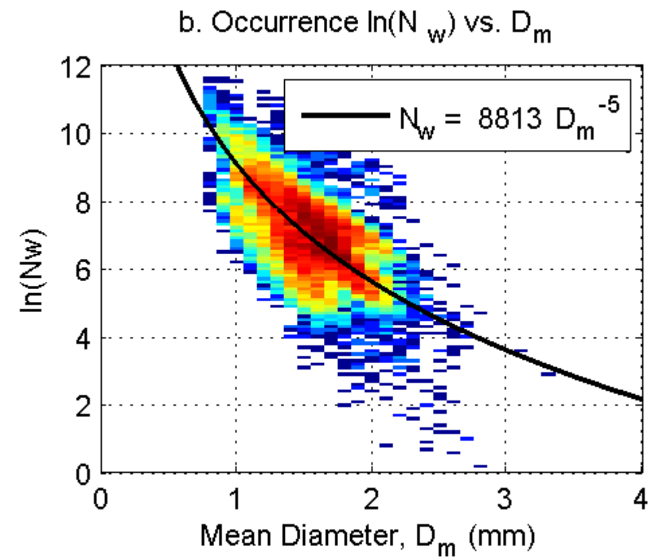


Frequency of Occurrences

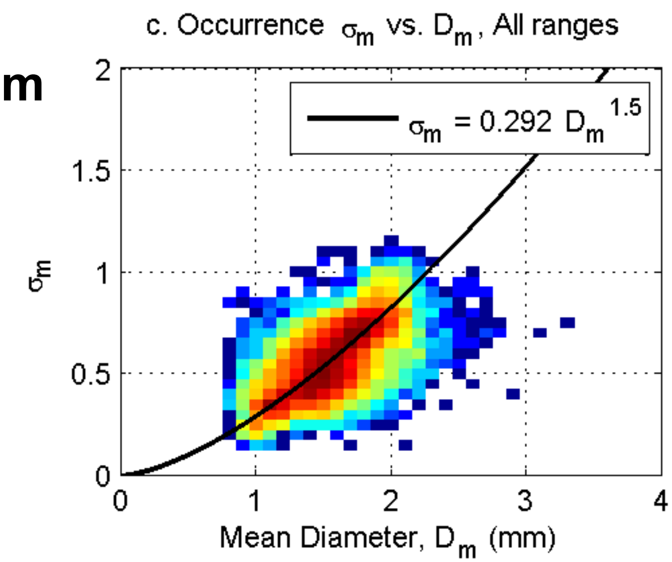
Z



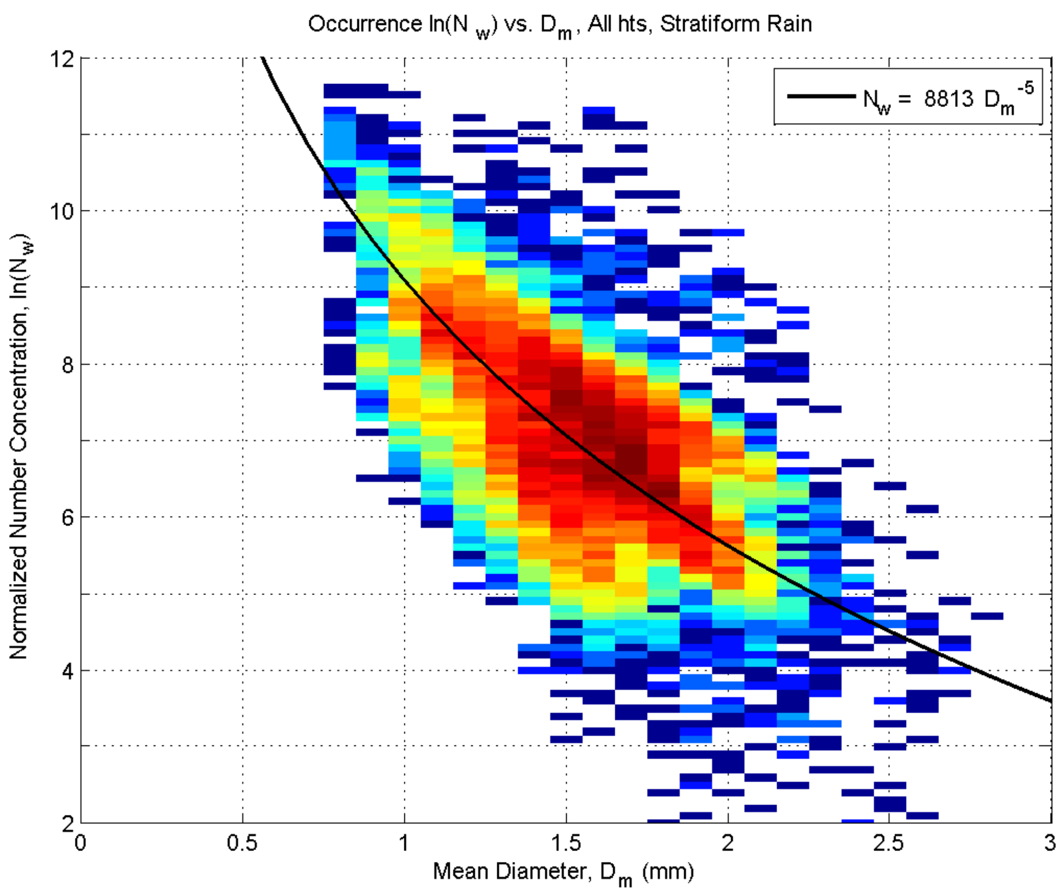
ln(Nw)



σ_m



Nw vs. Dm for all pixels (900 profiles x 24 heights)



This is all of the data.

The best fit has the form:

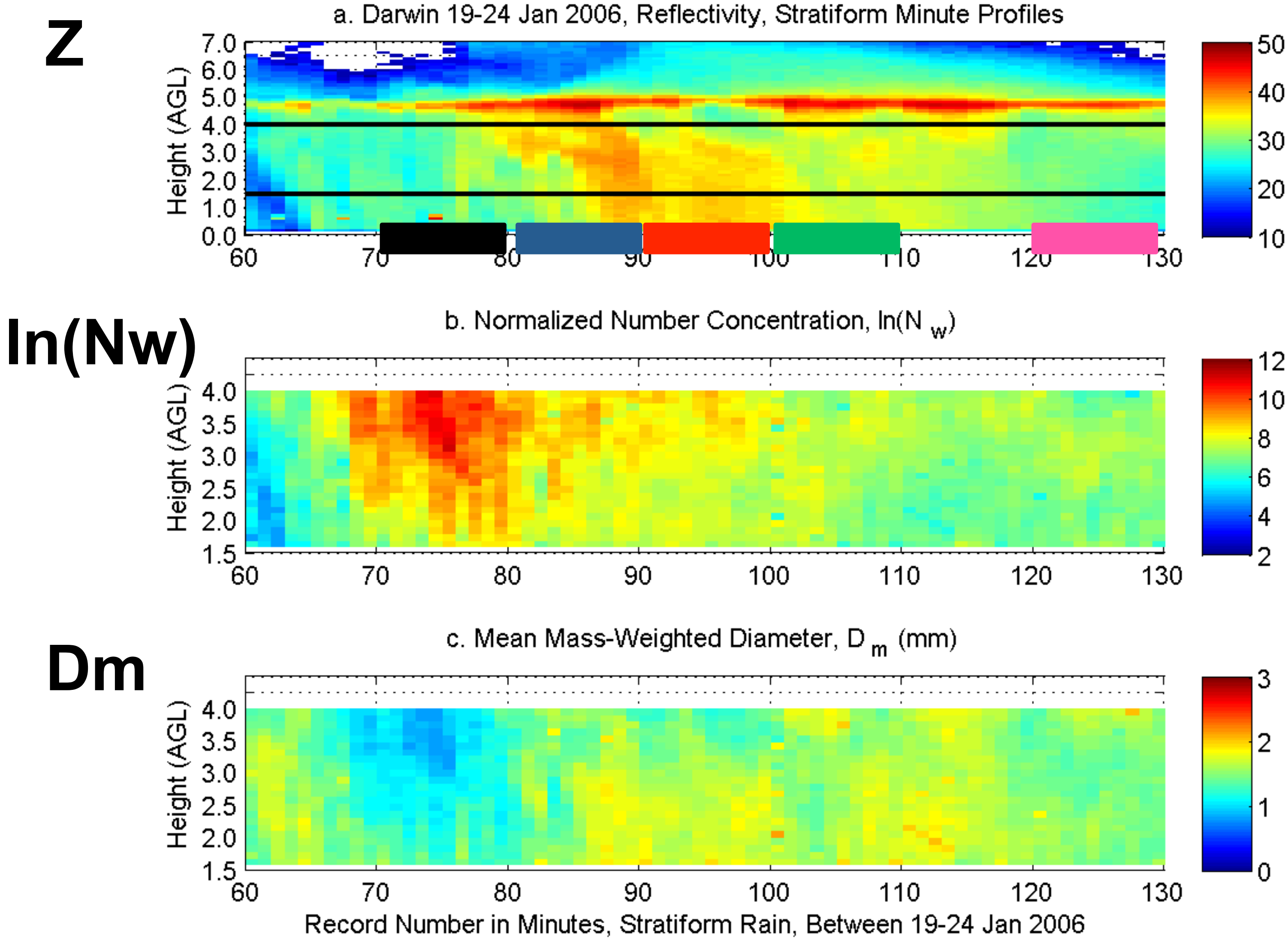
$$N_w = a_{Nw} D_m^{-5}$$

Can rearrange equation:

$$a_{Nw} = N_w D_m^5$$

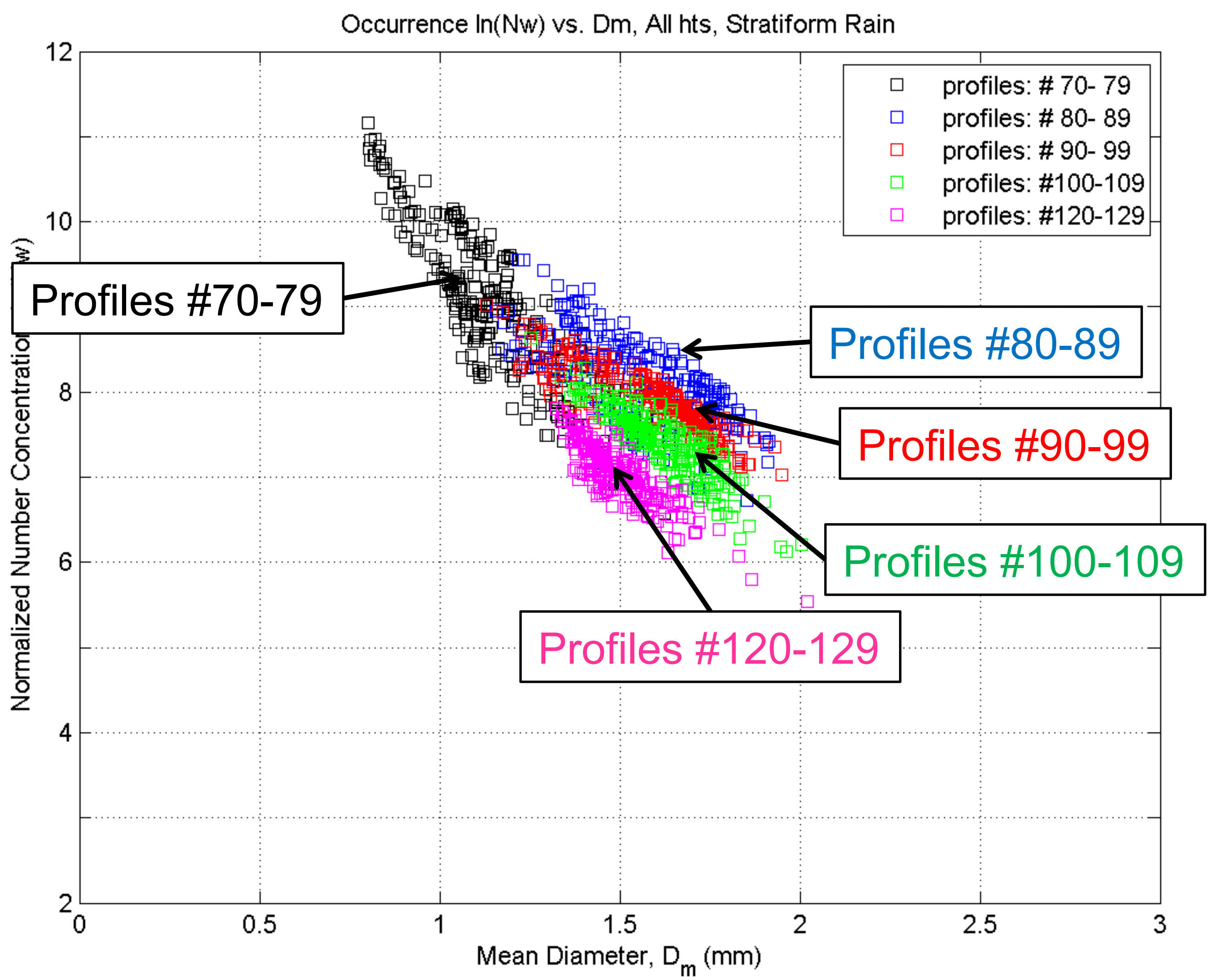
a_{Nw} is independent of Dm

70 Minutes of Profiler Z, Nw, and Dm



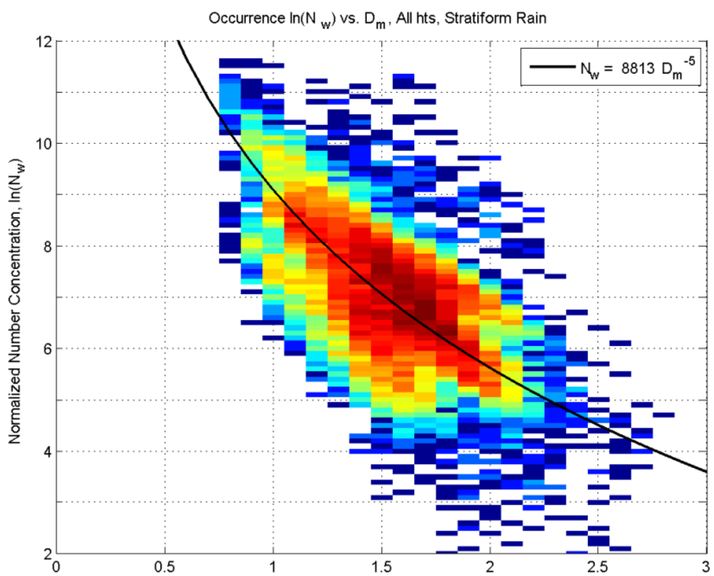
Colors show 10-minute time-height sections

N_w vs. D_m for 10-minute Sections



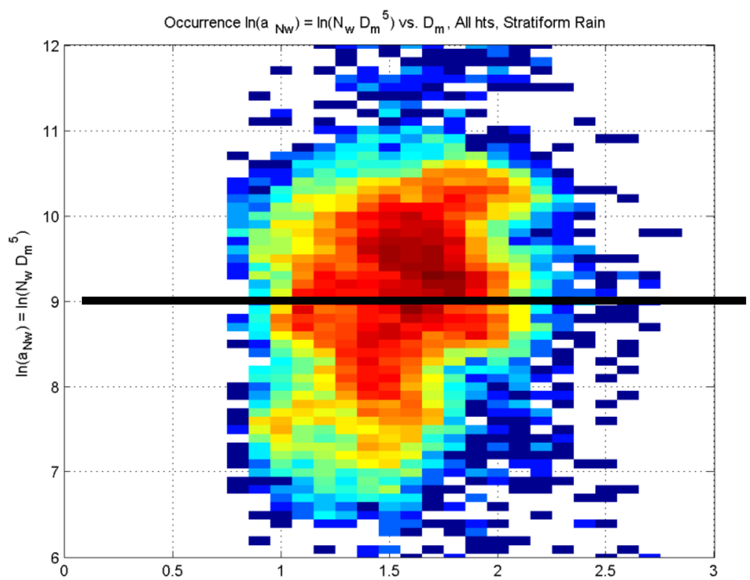
$a_{Nw} = N_w D_m^{5}$ vs. D_m

N_w vs. D_m



Diameter

$a_{Nw} = N_w D_m^5$ vs. D_m



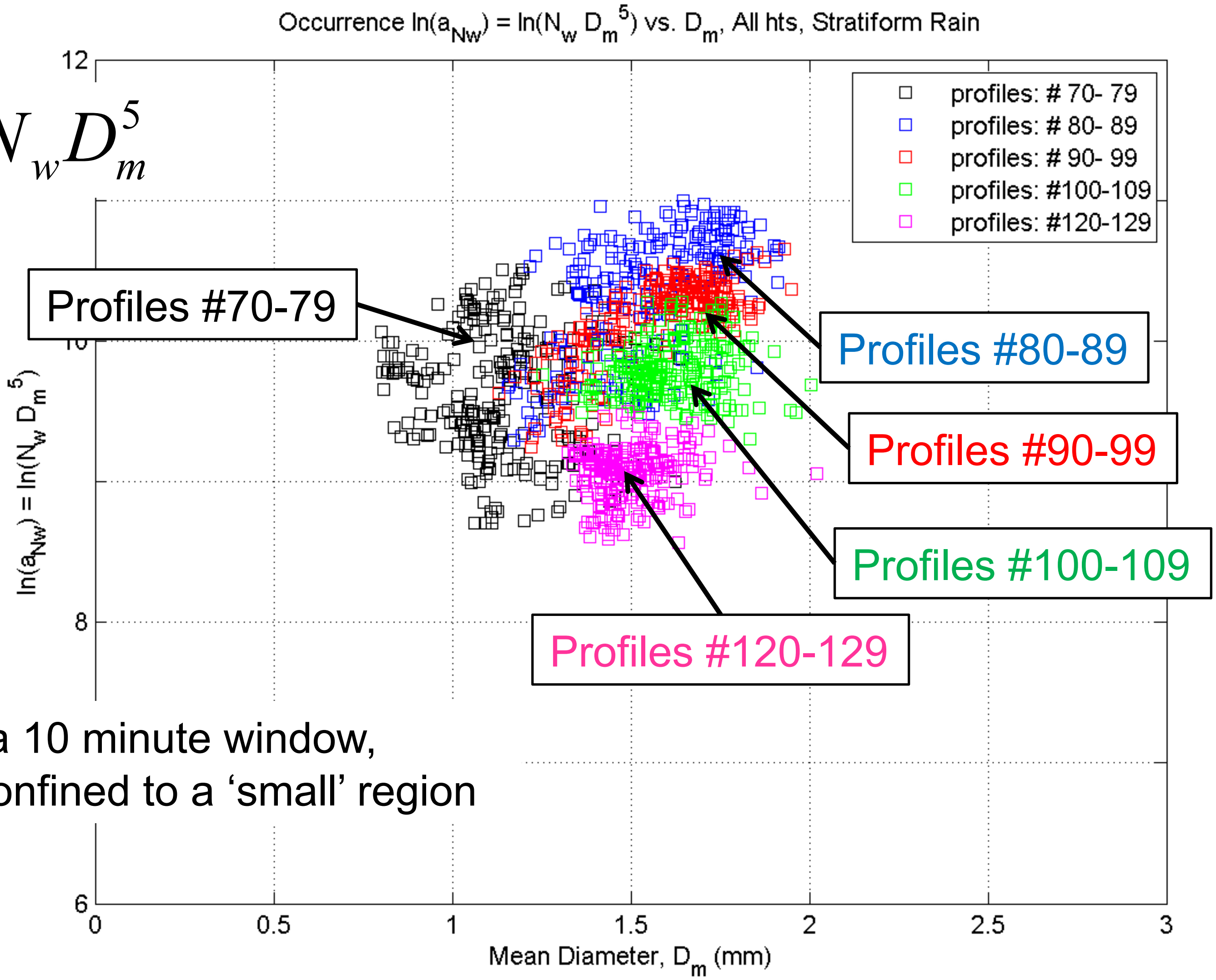
Diameter

a_{Nw} is independent of Dm

Wide range of a_{Nw}

$a_{Nw} = N_w D_m^5$ vs. D_m (10-minute Sections)

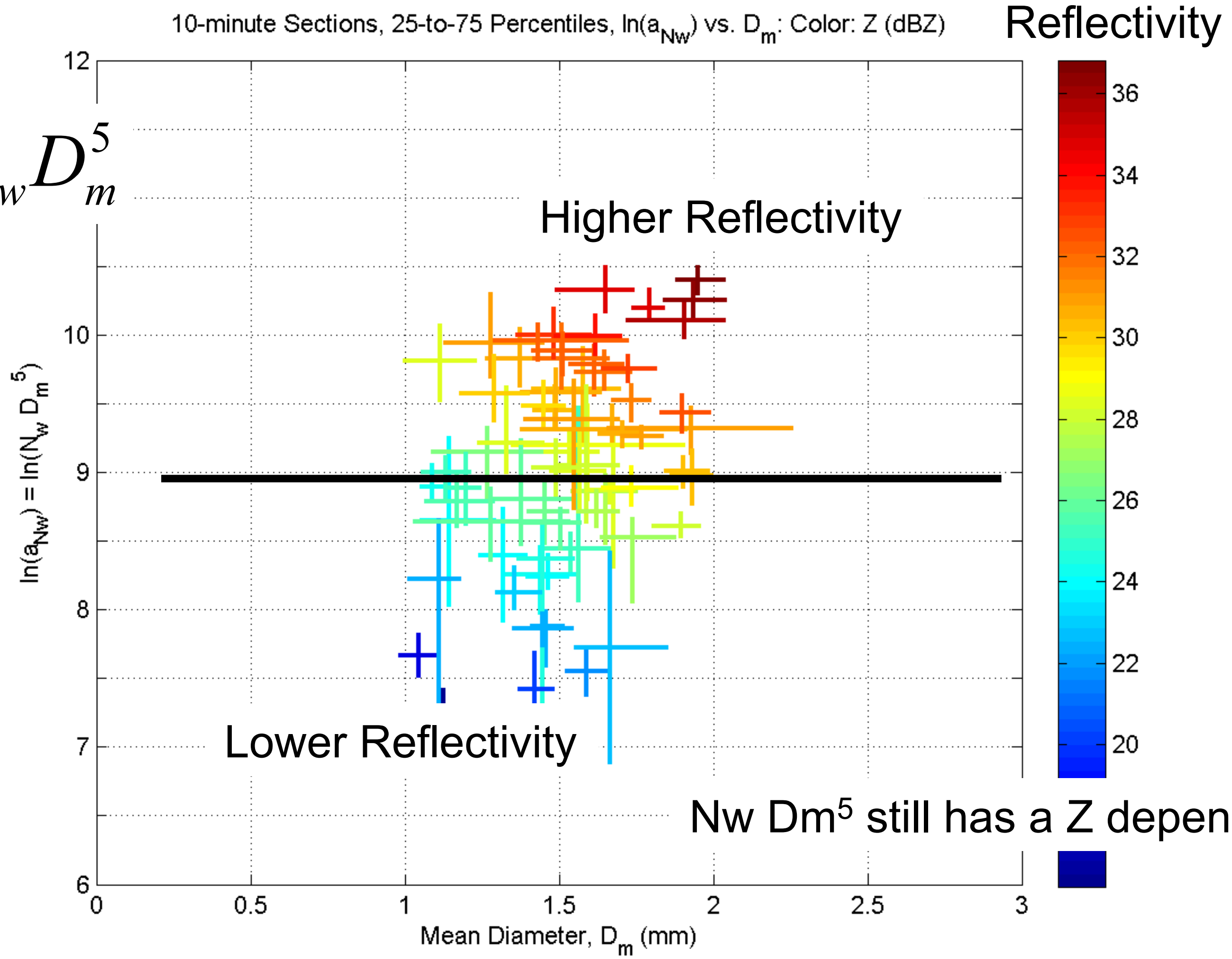
$$a_{Nw} = N_w D_m^5$$



During a 10 minute window,
a_{Nw} is confined to a 'small' region

$a_{Nw} = N_w D_m^5$ vs. D_m
all 10-minute sections (69 sections)

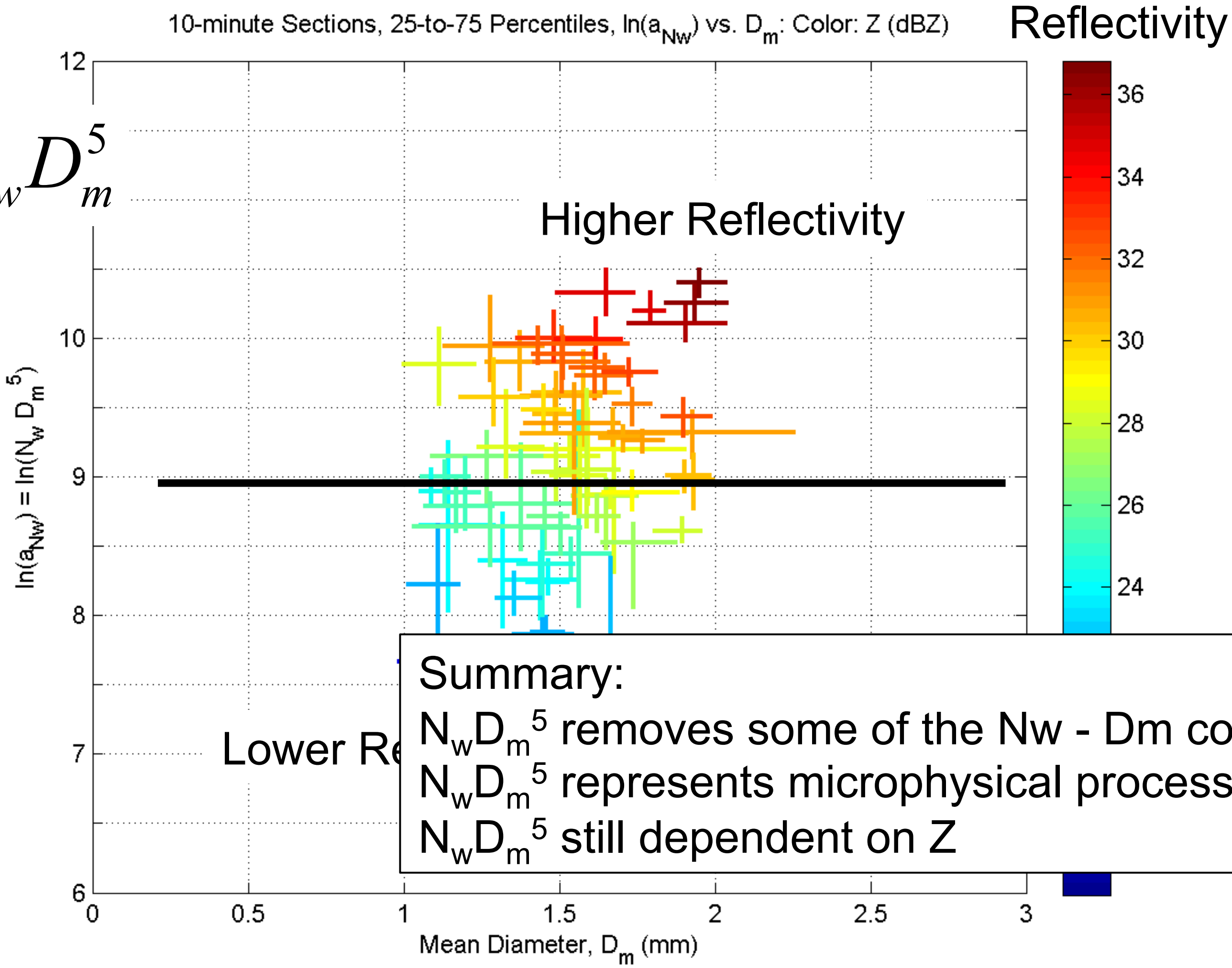
$$a_{Nw} = N_w D_m^5$$



Nw Dm^5 still has a Z dependence

$a_{Nw} = N_w D_m{}^5$ vs. D_m
all 10-minute sections (69 sections)

$$a_{Nw} = N_w D_m^5$$



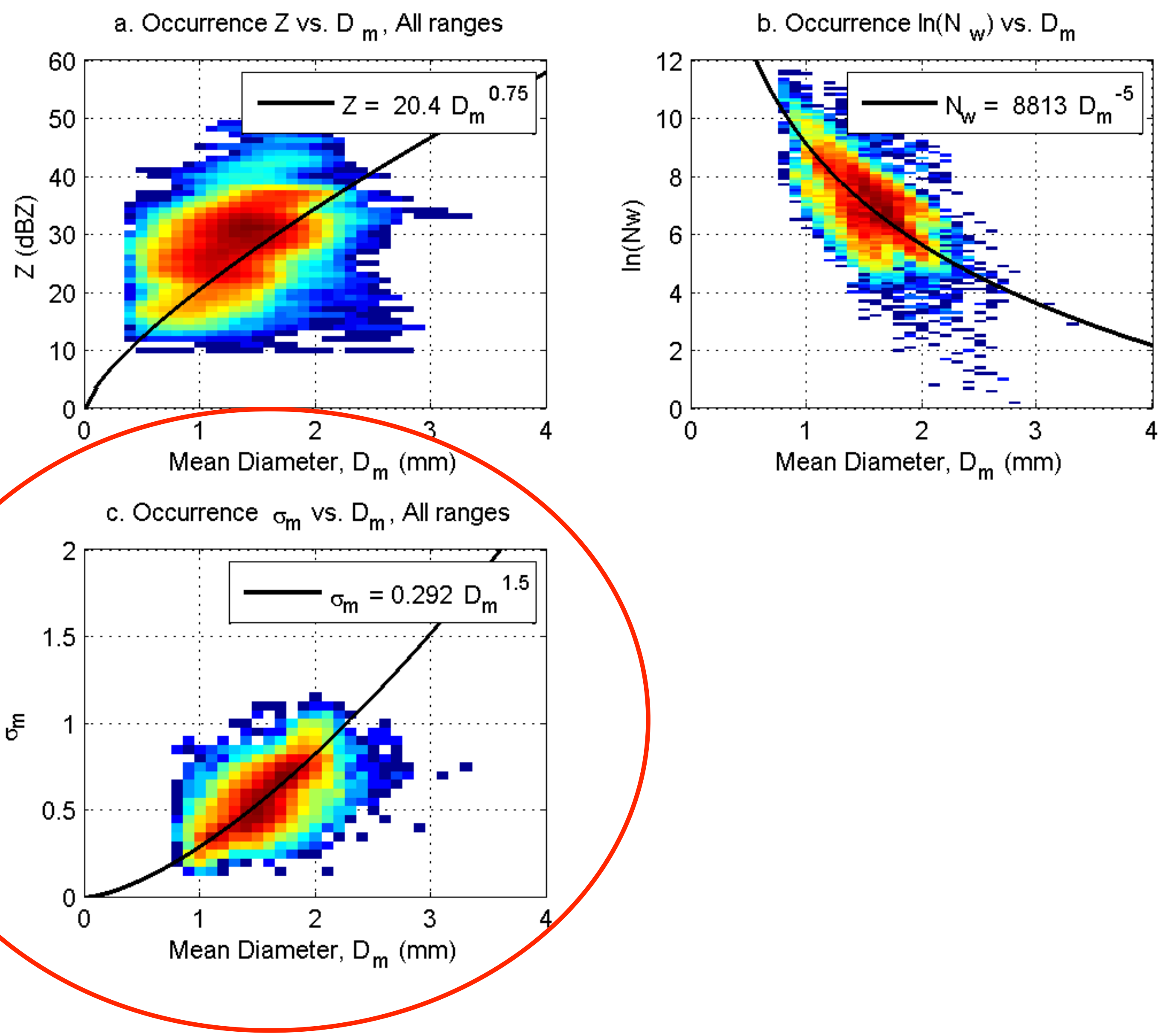
Summary:

$N_w D_m{}^5$ removes some of the Nw - Dm correlation

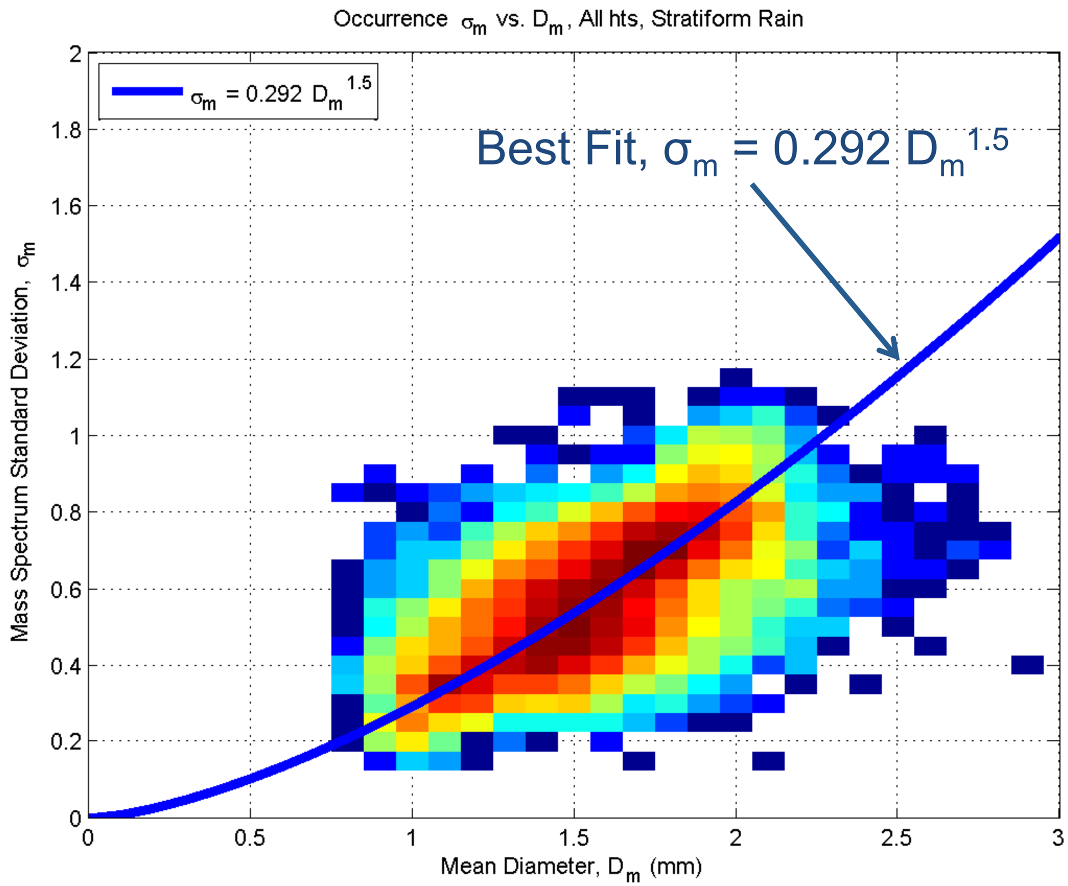
$N_w D_m{}^5$ represents microphysical processes

$N_w D_m{}^5$ still dependent on Z

Frequency of Occurrences

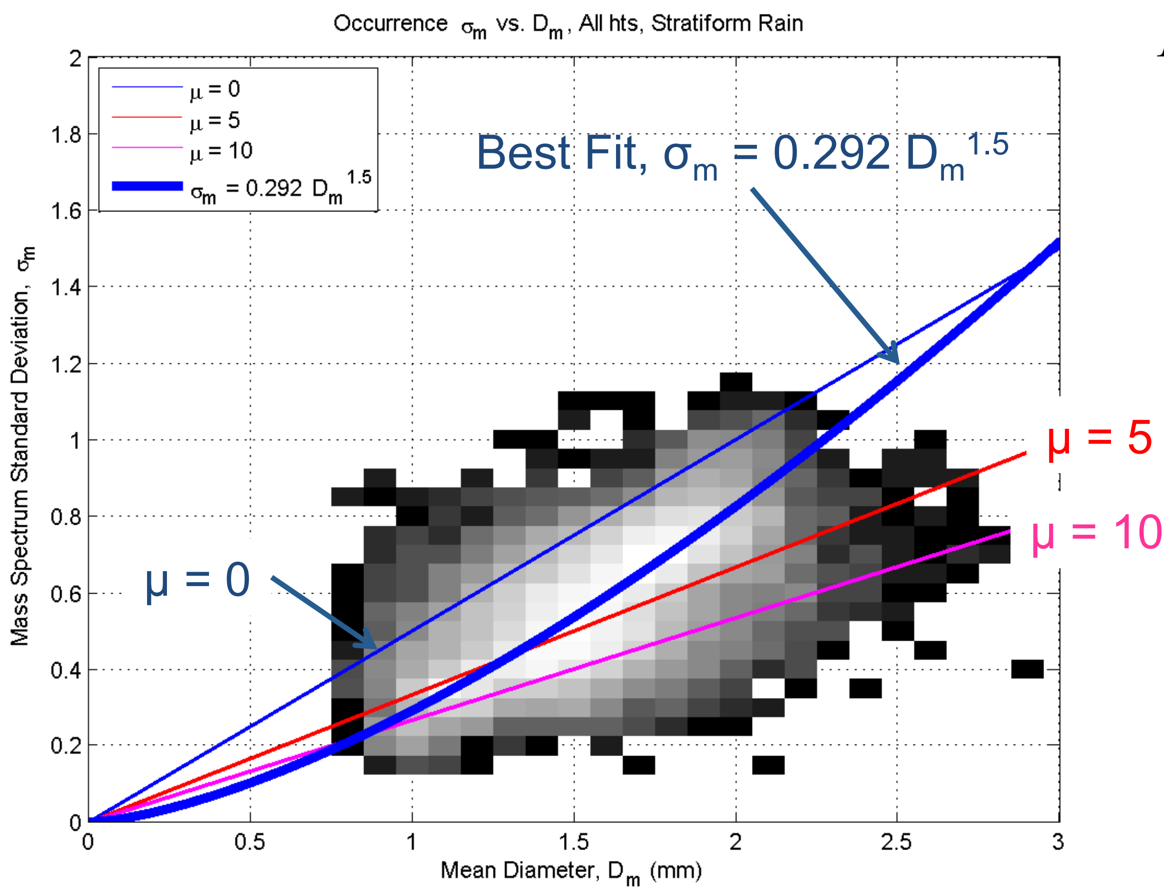


σ_m vs. D_m for all pixels

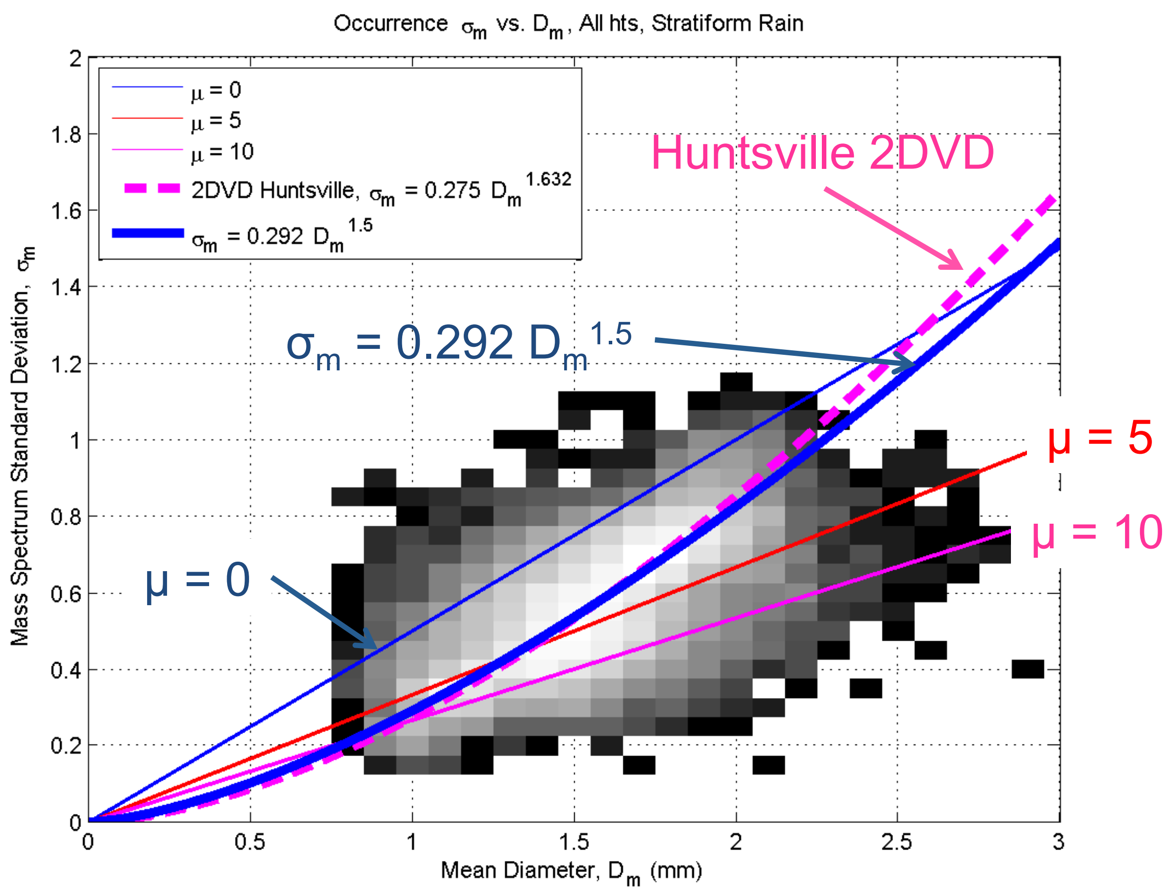


Plot σ_m vs. D_m for fixed values of μ

$$\frac{\sigma_m^2}{D_m^2} = \frac{1}{4+\mu}$$

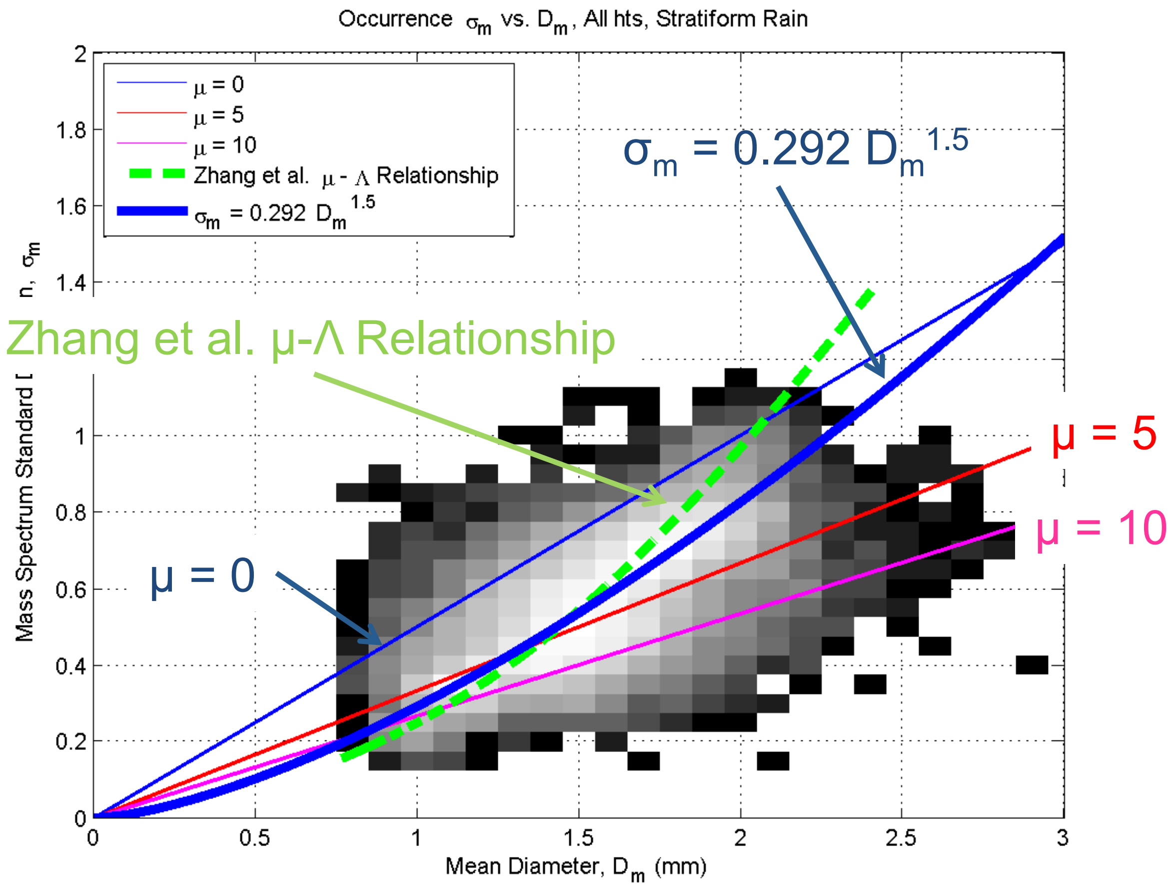


σ_m vs. D_m shape is robust – Darwin Profiler & Huntsville 2DVD

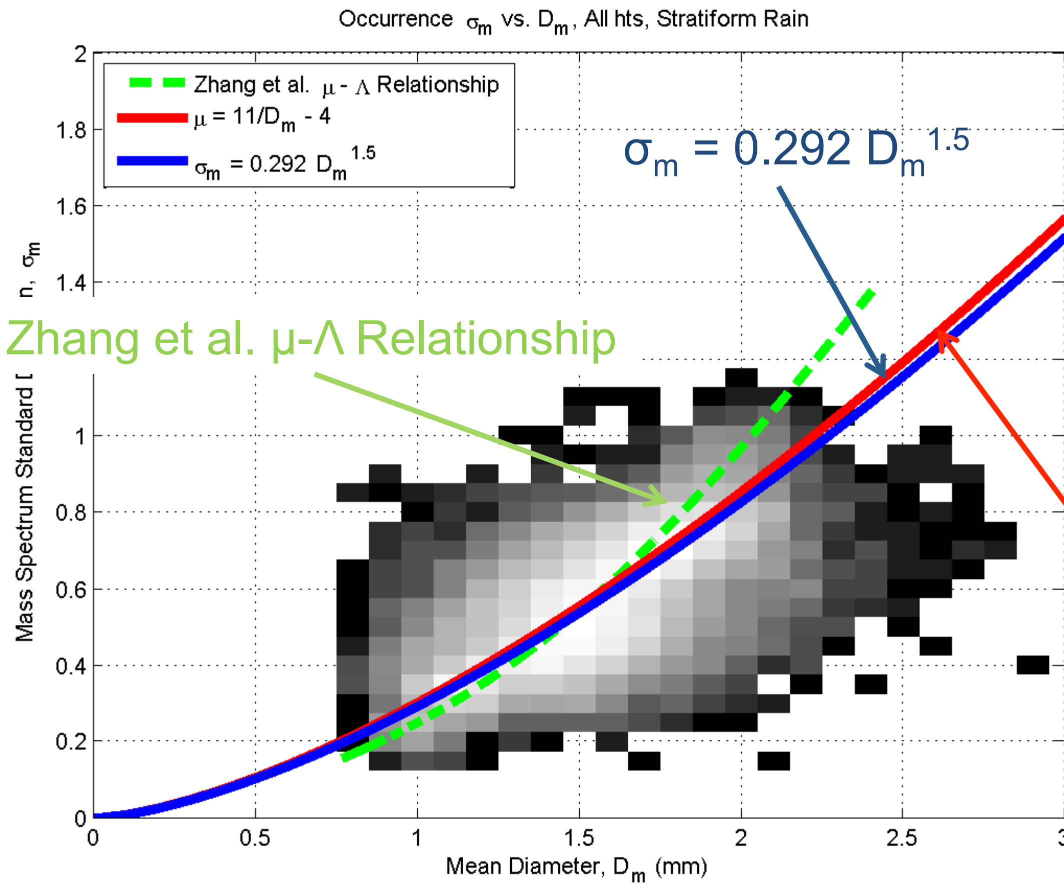


σ_m vs. D_m for all pixels
Zhang et al. μ-Λ Relationship

$$\Lambda = 0.365\mu^2 + 0.735\mu + 1.935$$



σ_m vs. D_m for all pixels Approximate μ - D_m Relationship



$$\frac{\sigma_m^2}{D_m^2} = \frac{1}{4+\mu}$$

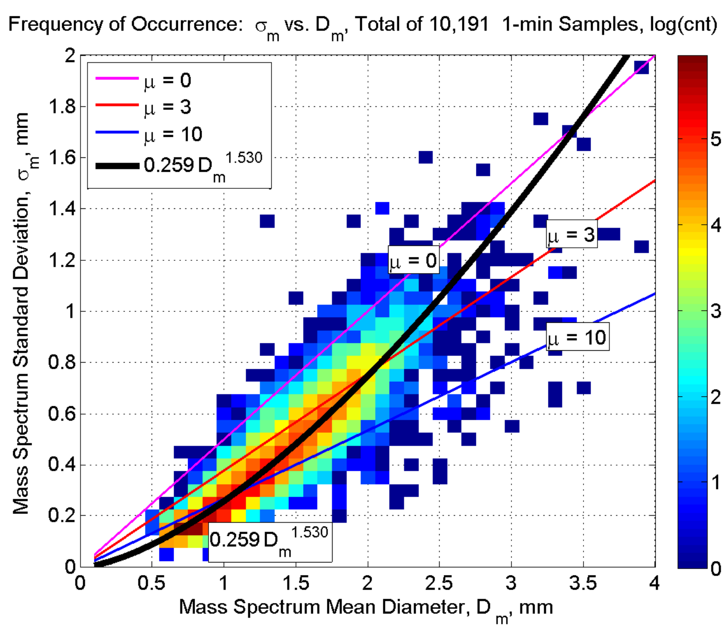
Let

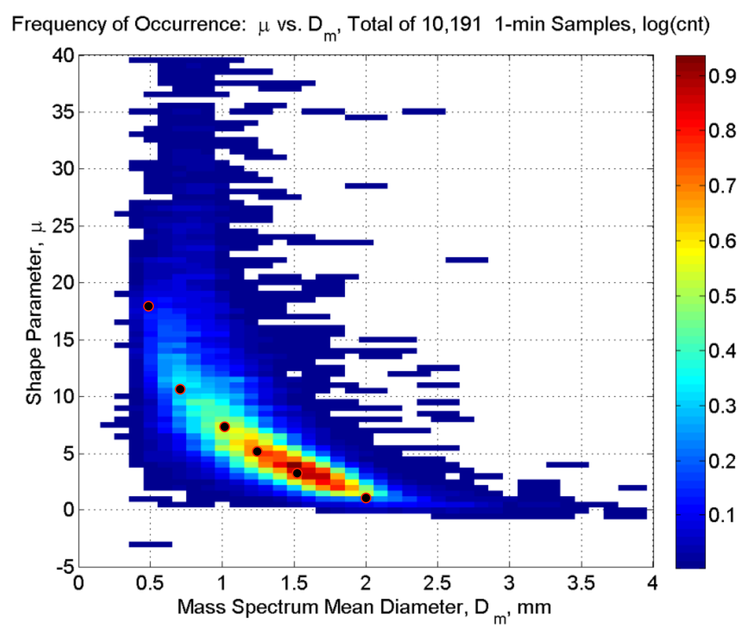
$$\sigma_m^2 \approx 0.09 D_m^3$$

$$0.09 D_m \approx \frac{1}{4+\mu}$$

$$\mu \approx \frac{11}{D_m} - 4$$

Frequency of Occurrence, σ_m vs. D_m Huntsville, 2DVD



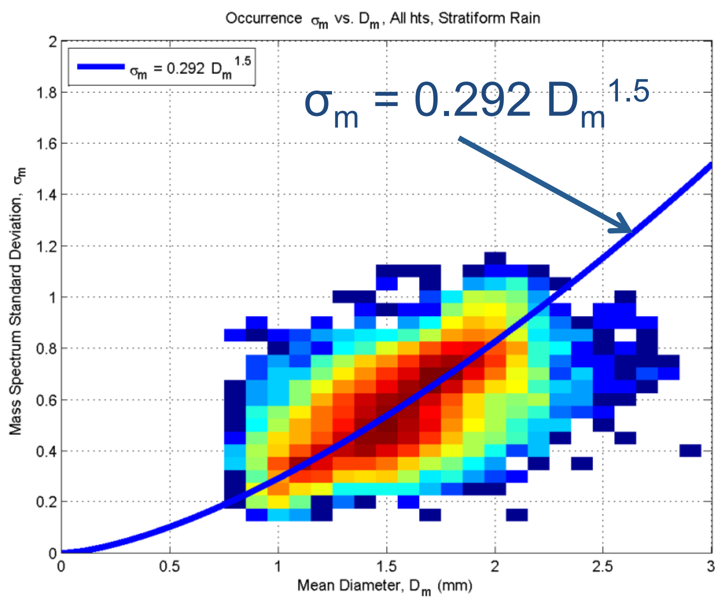


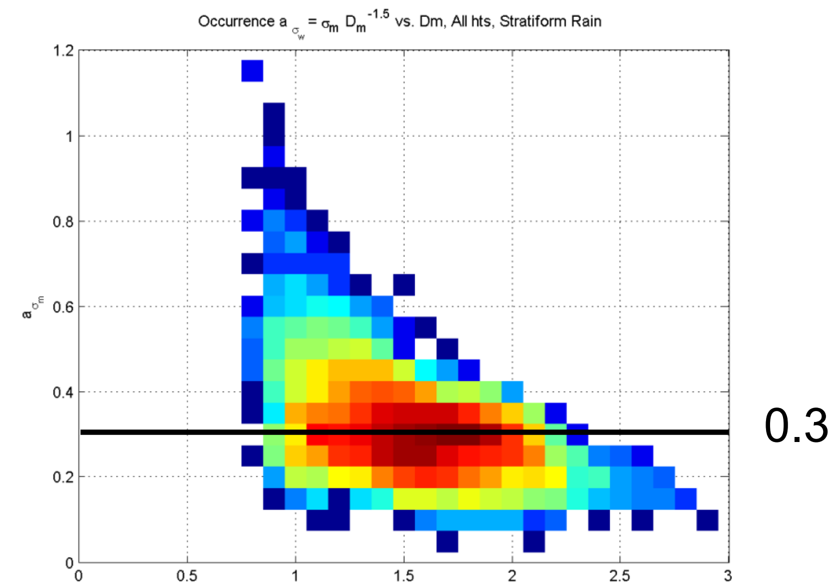
$$\mu \approx \frac{11}{D_m} - 4$$

σ_m vs. D_m and $a_{\sigma m} = \sigma_m D_m^{-1.5}$ vs. D_m

σ_m vs. D_m

$$a_{\sigma_m} = \sigma_m D_m^{-1.5}$$



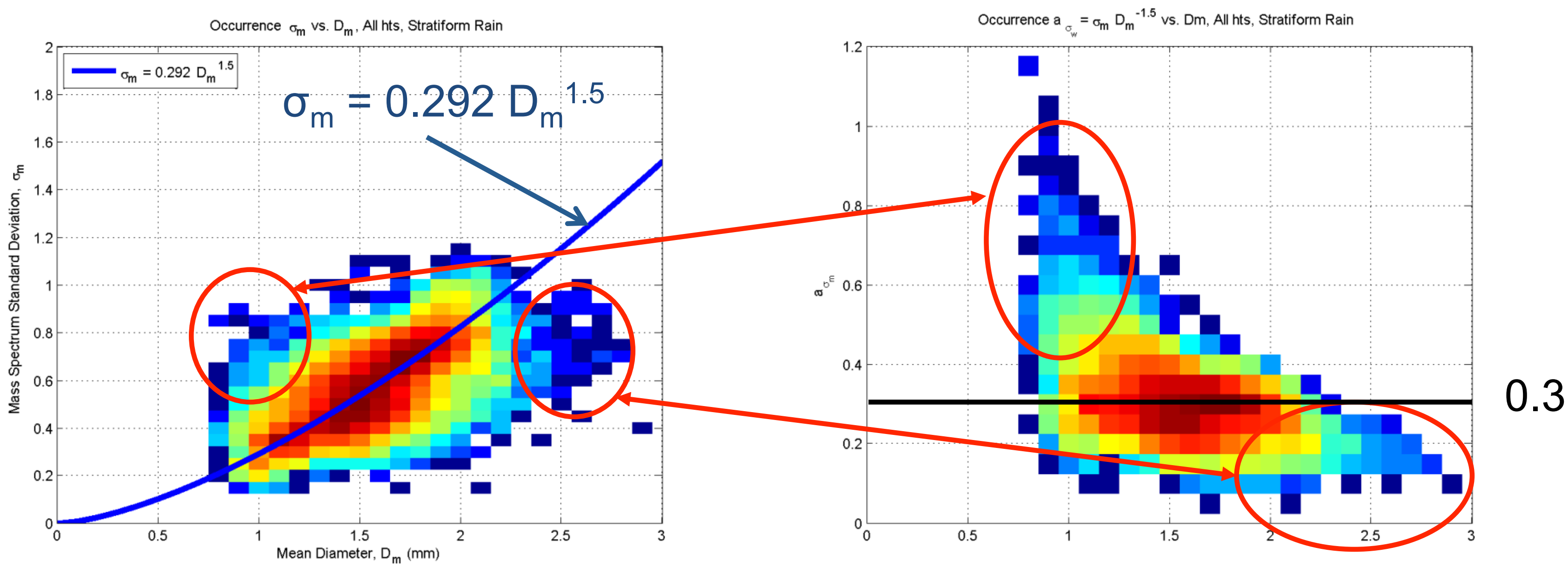


Colors are log(Occurrence) – few counts have color

σ_m vs. D_m and $a_{\sigma m} = \sigma_m D_m^{-1.5}$ vs. D_m

σ_m vs. D_m

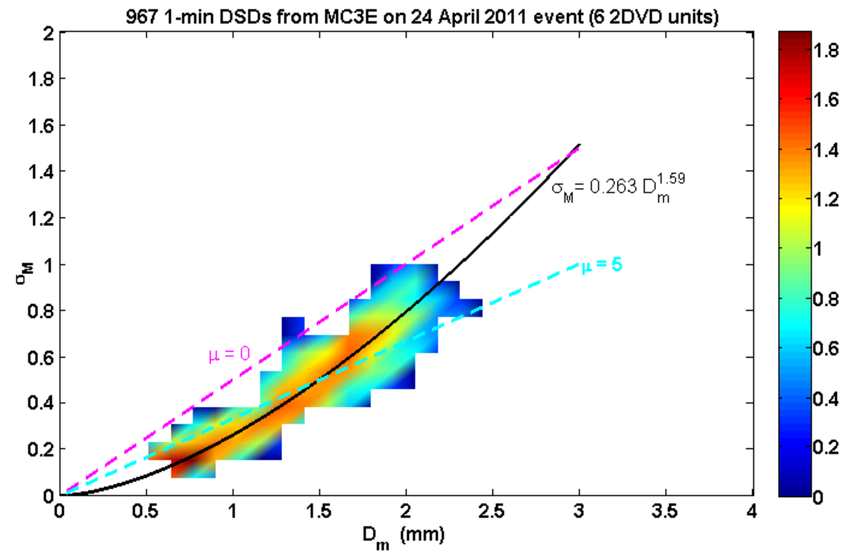
$a_{\sigma_m} = \sigma_m D_m^{-1.5}$



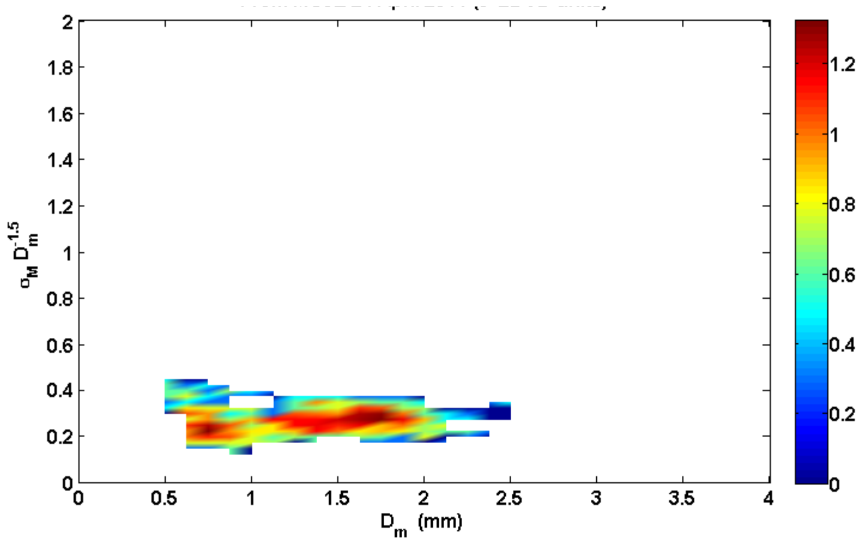
Colors are log(Occurrence) – even few counts have color

Extreme values go away when aggregating 10-minute sections

Moving Forward: Frequency of Occurrence: σ_m vs. D_m MC3E – Six 2DVD units – 24 April 2011



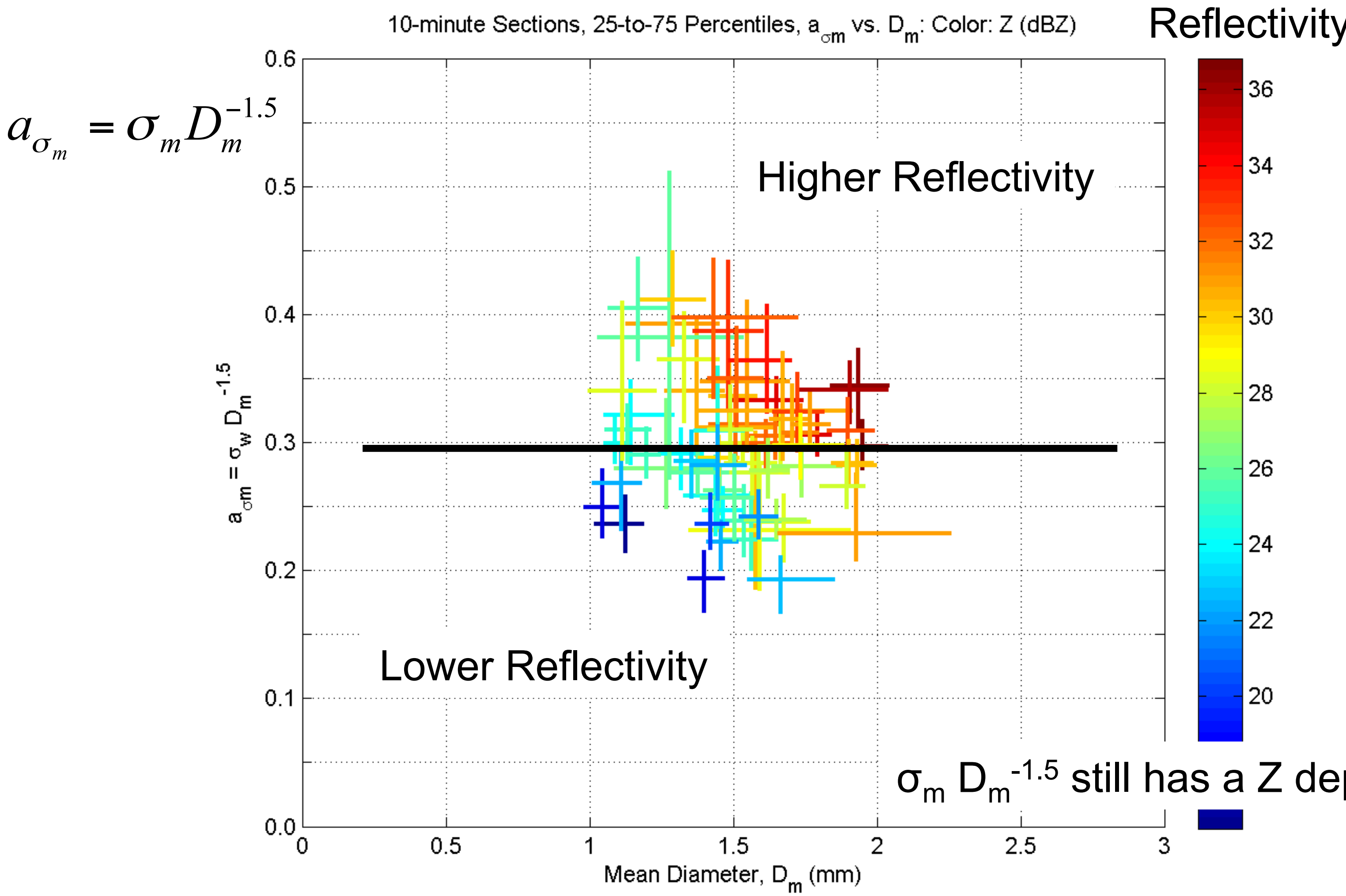
Bringi's analysis:
σ_m vs. D_m Relationship appears robust



Normalized variable:
$\sigma_m D_m^{-1.5}$ vs. Dm

Vertical variation of $\sigma_m D_m^{-1.5}$ is useful for algorithms by setting bounds on expected range

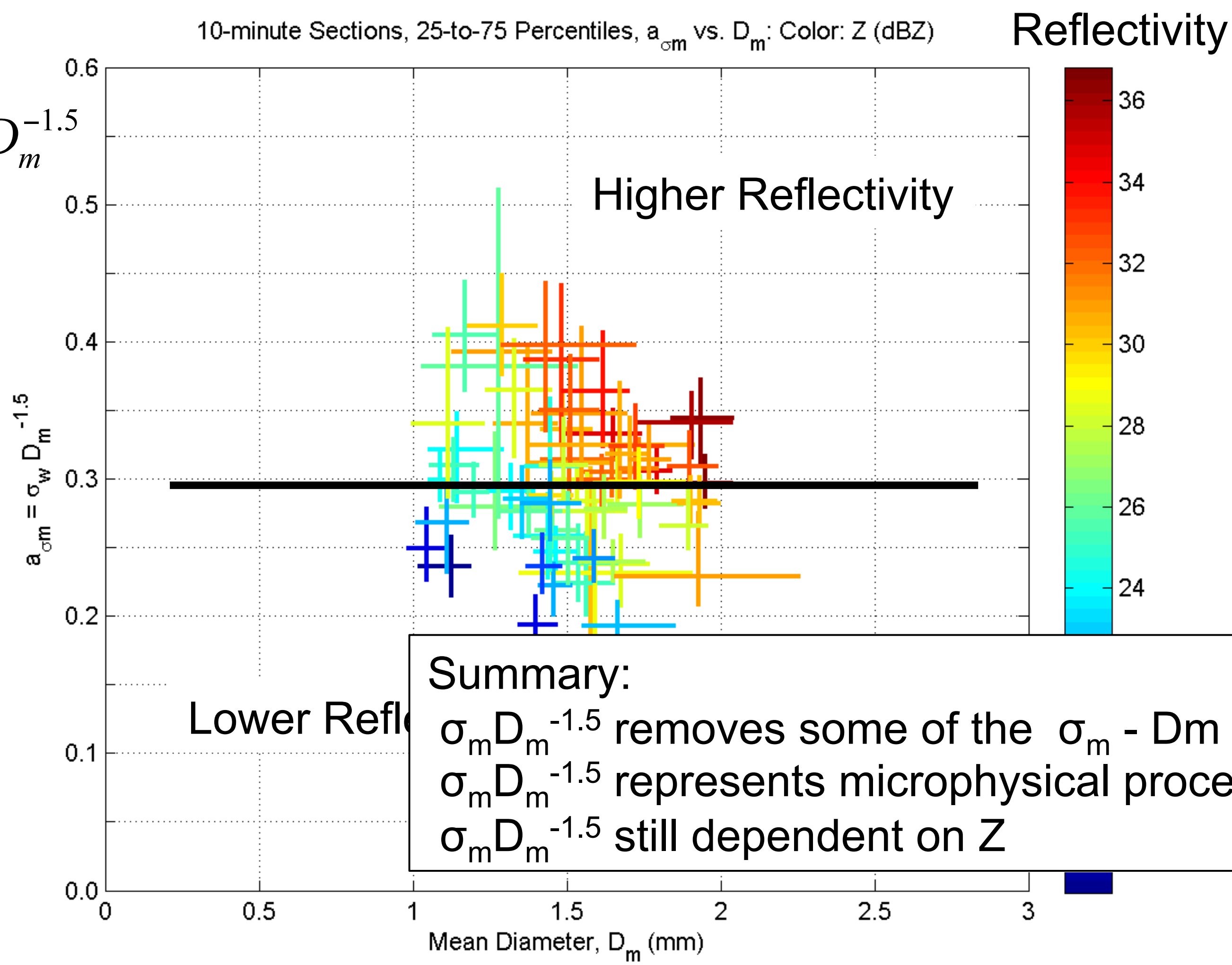
$a_{\sigma m} = \sigma_m D_m^{-1.5}$ vs. D_m
10-minute Sections (69 sections)



$\sigma_m D_m^{-1.5}$ still has a Z dependence

$a_{\sigma m} = \sigma_m D_m^{-1.5}$ vs. D_m
10-minute Sections (69 sections)

$$a_{\sigma_m} = \sigma_m D_m^{-1.5}$$



Summary:

$\sigma_m D_m^{-1.5}$ removes some of the σ_m - Dm correlation
$\sigma_m D_m^{-1.5}$ represents microphysical processes
$\sigma_m D_m^{-1.5}$ still dependent on Z

Concluding Remarks

- DSD Working Group is charged to investigate correlations between DSD parameters using GV data sets that support, or guide, the assumptions used in satellite retrieval algorithms.

- As guided by algorithm developers, normalizing N_w and σ_m by power-law relationships removes correlations with Dm

$$N_w = a_{Nw} D_m^{-5} \longrightarrow a_{Nw} = N_w D_m^{5}$$

$$\sigma_m = a_{\sigma_m} D_m^{1.5} \longrightarrow a_{\sigma_m} = \sigma_m D_m^{-1.5}$$

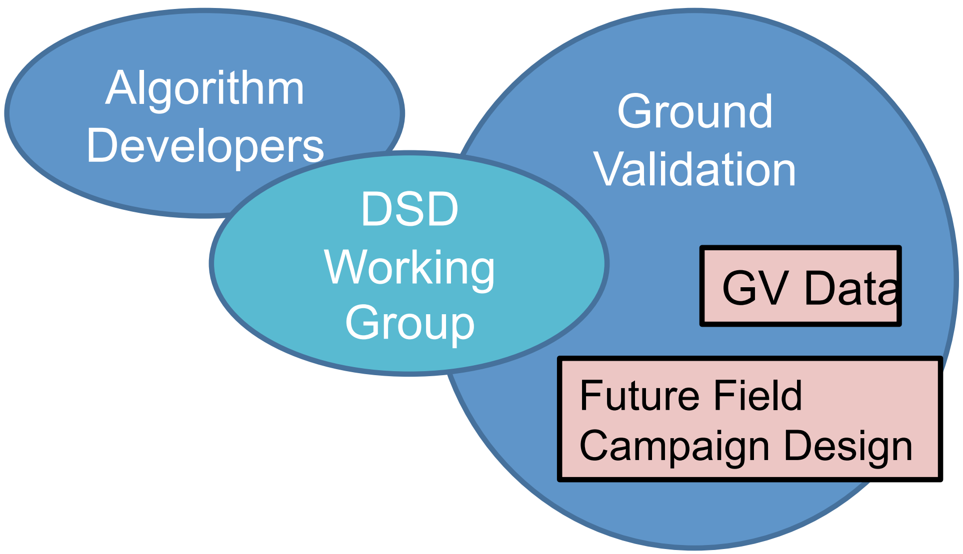
- Normalized coefficients a_{Nw} and $a_{\sigma m}$ in power law relations:
  - may show regime dependent signatures
  - Narrow range of values in time-height sections
  - Are still Z dependent
- σ_m – Dm relationship is robust – use this if you can
- If you need to use μ, maybe a μ - D_m relationship may be better than a constant value. To first order, GV data suggests something of the form:

$$\mu \approx \frac{11}{D_m} - 4$$

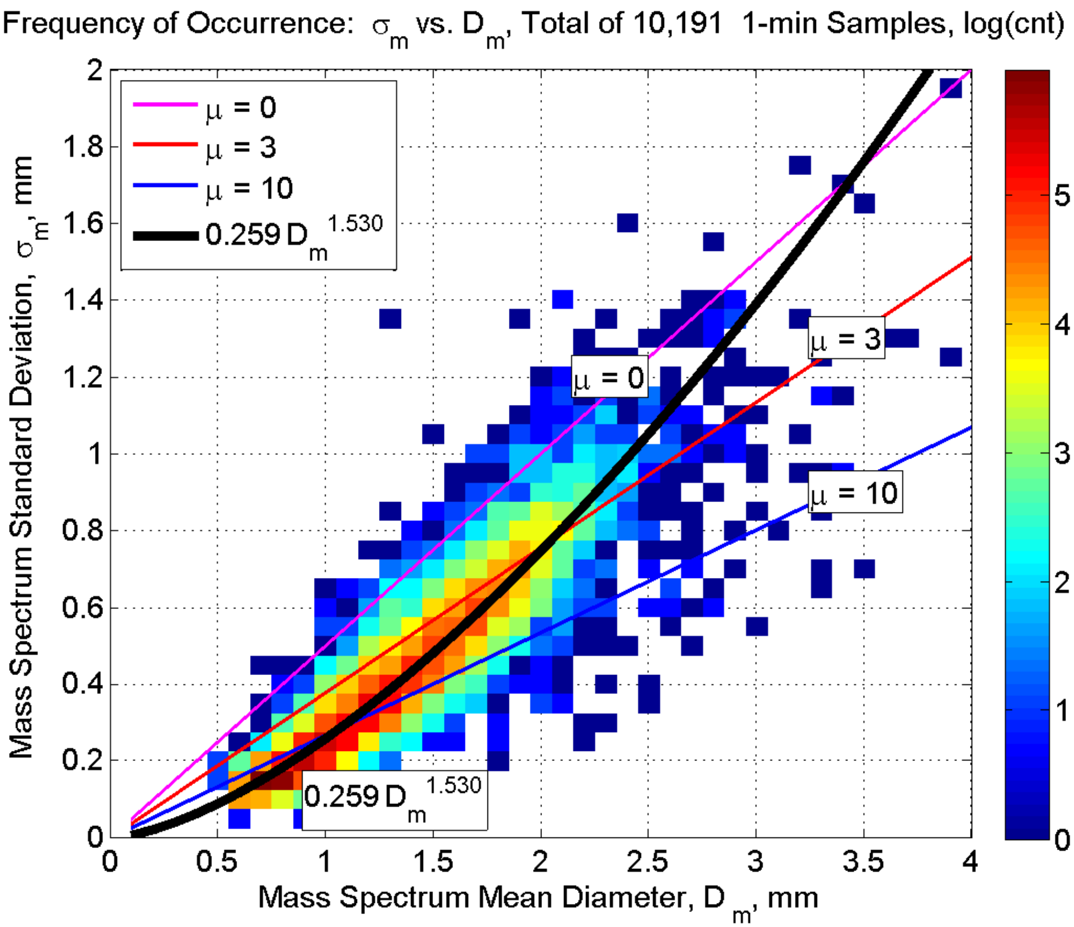
Next Steps

- DSD Working Group needs input and guidance from algorithm developers on what assumptions they want validated or supported with GV data.

- DSD Working Group can help with boundaries or assumptions used in scattering tables
  - Example: D_{max} in Integral tables – D_{max} = 8 mm or $D_{max} = 3D_m$?



Frequency of Occurrence σ_m vs. D_m



- σ_m increases as D_m increases
- Best fit line represents the most frequent
  - $\sigma_m = 0.259\ D_m^{1.53}$
- Gamma DSDs have this relationship:
  - $\sigma_m^2 / D_m^2 = 1/(4+\mu)$
- Lines show σ_m vs. D_m for Gamma DSDs with μ = 0, 3, & 10.

Percent Variance Explained by EOFs

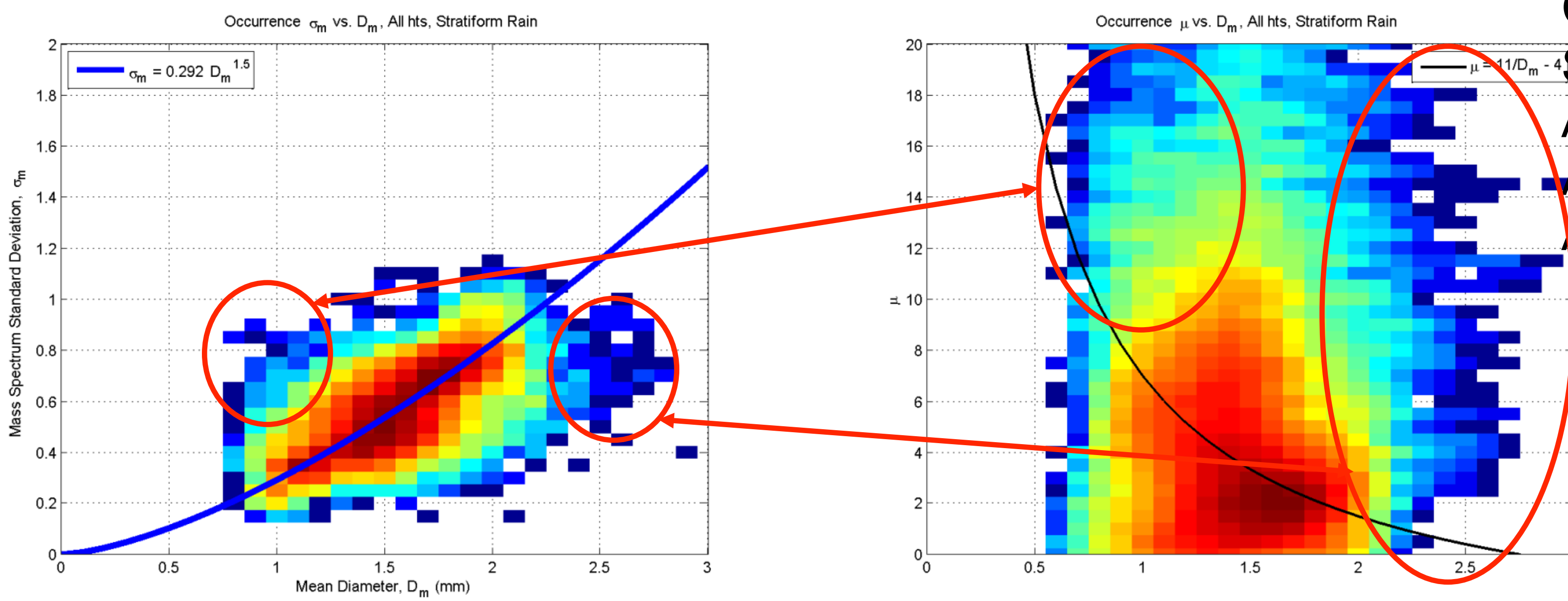
	Z	Nw	Dm	σ_m	μ	a_{Nw}	$a_{\sigma m}$
	71.8577	37.058	41.7681	38.8766	25.0006	68.2073	24.818
87%	15.7153	14.5051	14.2694	14.42	11.63	13.5825	12.7418
	5.296	11.0523	7.9011	6.8408	7.8723	5.0166	7.9605
	2.0713	8.6653	5.2159	5.1612	5.7965	2.6294	5.8422
	1.1172	5.048	3.8345	3.9474	4.9905	1.7925	4.8058
	0.68517	3.9922	3.0303	3.0722	4.7138	1.4376	4.3918
	0.48292	3.6856	2.5152	2.4987	4.5162	1.1167	3.9067
	0.35196	2.4061	2.173	2.2993	3.7775	1.0244	3.4929
	0.26153	2.1977	1.8537	2.1274	3.5254	0.61621	3.2365
	0.23147	1.7773	1.8232	2.0671	3.245	0.57868	2.9058
	0.20539	1.392	1.7043	1.8608	2.8172	0.47105	2.7648
	0.18983	1.2674	1.5784	1.7922	2.7102	0.40913	2.5951
	0.17262	1.0212	1.5545	1.6876	2.6193	0.38918	2.5373
	0.16996	0.97983	1.3956	1.6456	2.2905	0.36857	2.3432
	0.16308	0.94374	1.2671	1.559	2.2416	0.34064	2.2707
	0.1496	0.77287	1.2381	1.5424	2.0545	0.31755	2.0605
	0.14655	0.68588	1.2	1.4155	1.8976	0.31229	2.0047
	0.13442	0.67091	1.159	1.3444	1.7461	0.28485	1.8203
	0.13327	0.54703	1.0389	1.2874	1.6544	0.25844	1.7257
	0.12671	0.41171	0.97321	1.2508	1.5103	0.25047	1.6163
	0.1209	0.37613	0.94364	1.1769	1.3266	0.2274	1.5242
	0.11046	0.28893	0.83013	1.1356	1.1336	0.20054	1.3574
	0.10676	0.25474	0.73259	0.991	0.93048	0.16818	1.2778
	1.0e-027	4.3e-030	2.6e-028	2.2e-028	8.1e-030	2.1e-027	5.5e-029

Z eign	**ln(Nw)**	Dm eign	Sm eign	mu eign	**ln(a_Nw)** eign	a_Sm eign
71.8577	58.8782	41.7681	38.8766	25.0006	68.2073	24.818
15.7153	9.9842	14.2694	14.42	11.63	13.5825	12.7418
5.296	4.898	7.9011	6.8408	7.8723	5.0166	7.9605
2.0713	3.043	5.2159	5.1612	5.7965	2.6294	5.8422
1.1172	2.3906	3.8345	3.9474	4.9905	1.7925	4.8058
0.68517	1.854	3.0303	3.0722	4.7138	1.4376	4.3918
0.48292	1.6735	2.5152	2.4987	4.5162	1.1167	3.9067
0.35196	1.448	2.173	2.2993	3.7775	1.0244	3.4929
0.26153	1.3591	1.8537	2.1274	3.5254	0.61621	3.2365
0.23147	1.289	1.8232	2.0671	3.245	0.57868	2.9058
0.20539	1.222	1.7043	1.8608	2.8172	0.47105	2.7648
0.18983	1.1992	1.5784	1.7922	2.7102	0.40913	2.5951
0.17262	1.1228	1.5545	1.6876	2.6193	0.38918	2.5373
0.16996	1.0892	1.3956	1.6456	2.2905	0.36857	2.3432
0.16308	1.0625	1.2671	1.559	2.2416	0.34064	2.2707
0.1496	1.0267	1.2381	1.5424	2.0545	0.31755	2.0605
0.14655	0.99398	1.2	1.4155	1.8976	0.31229	2.0047
0.13442	0.94017	1.159	1.3444	1.7461	0.28485	1.8203
0.13327	0.87154	1.0389	1.2874	1.6544	0.25844	1.7257
0.12671	0.79954	0.97321	1.2508	1.5103	0.25047	1.6163
0.1209	0.76516	0.94364	1.1769	1.3266	0.2274	1.5242
0.11046	0.74652	0.83013	1.1356	1.1336	0.20054	1.3574
0.10676	0.686	0.73259	0.991	0.93048	0.16818	1.2778
1.0744e-027	0.65693	2.6208e-028	2.2469e-028	8.1282e-030	2.1893e-027	5.5853e-029

Define some DSD Parameters (3/3)

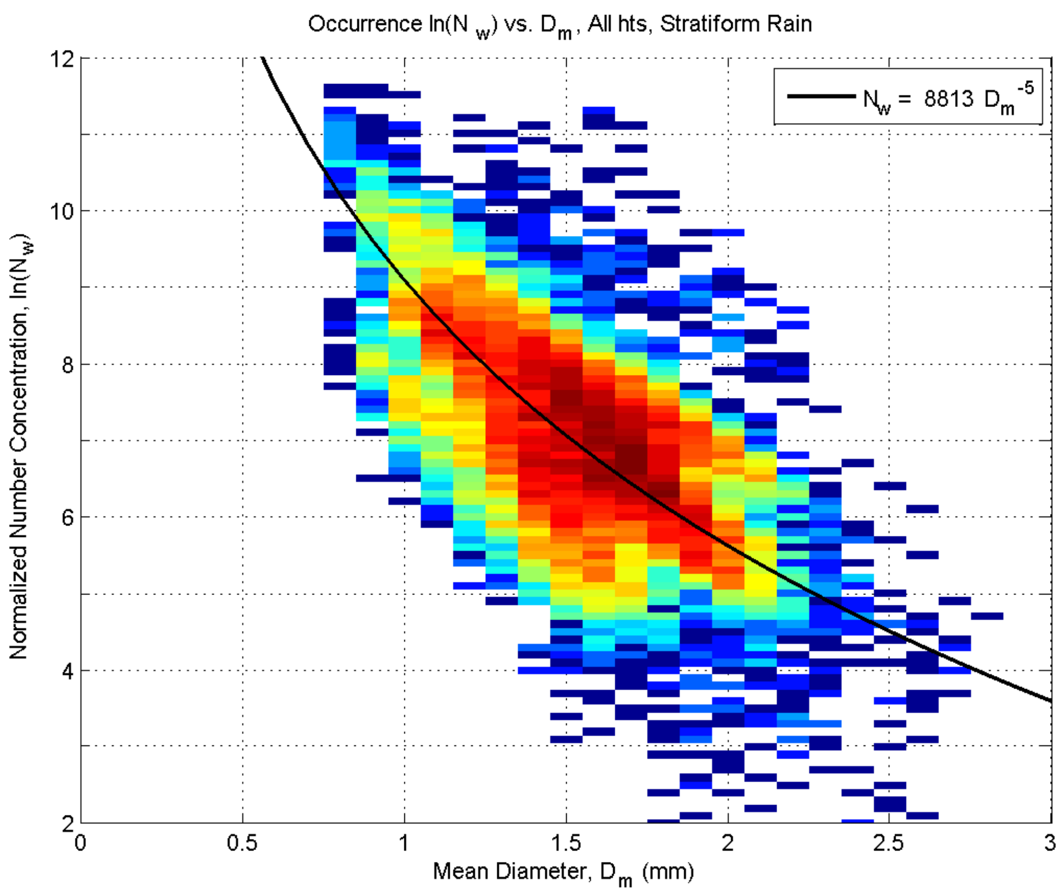
- Normalized number concentration
  - Nw = constant (W/Dm^4)
  - Nw has units of mm^-1
  - Nw = N_0* using Testud et al. (2001) notation
- *Conceptually*, the N(D) can be described:
  - N(D) ~ Nt pdf(D)
    - Where Nt is the total number of drops per unit volume and pdf(D) is the normalized DSD shape

σ_m vs. Dm for all pixels (4/4)



Change color
Show best fit
Add u lines
Add Zhang et
Add Bringi's H

Nw vs. Dm for all pixels (900 profiles x 24 heights)



This is all of the data.

The best fit has the form:

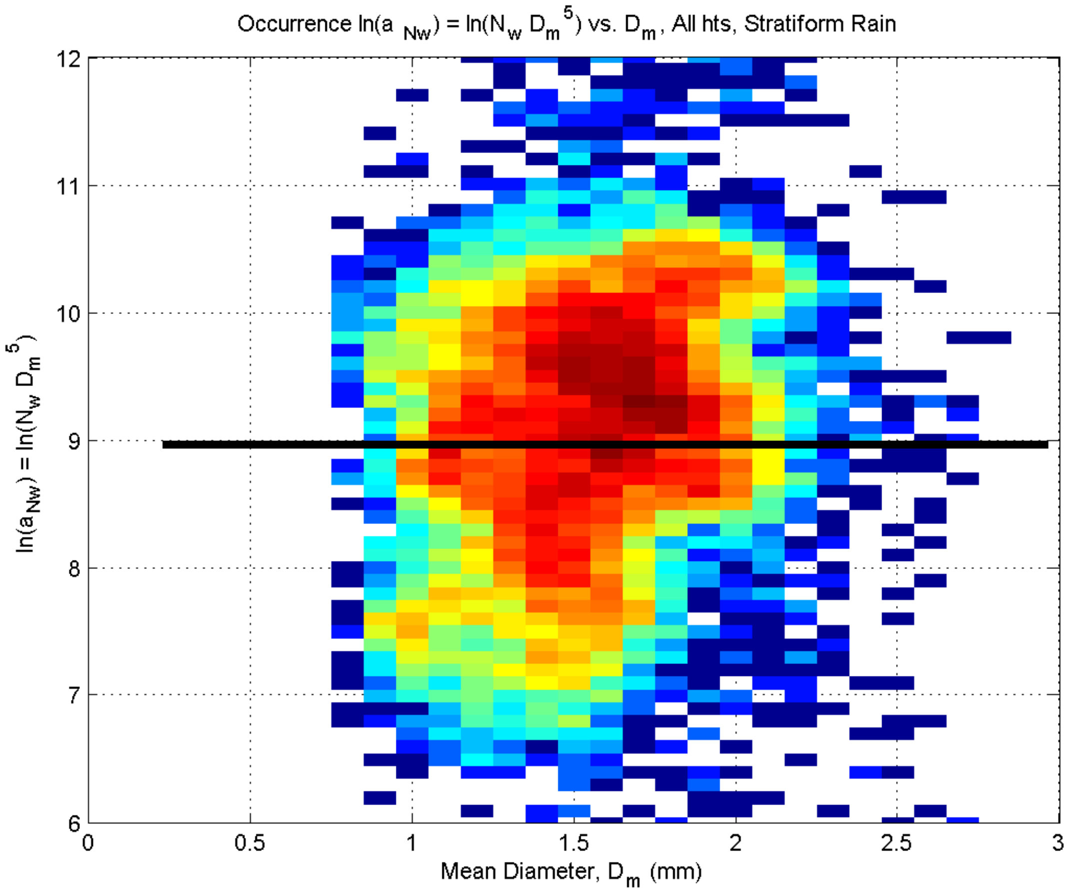
$$N_w = a_{Nw} D_m^{-5}$$

Can rearrange equation:

$$a_{Nw} = N_w D_m^{5}$$

a_{Nw} tries to remove the Correlation with Dm

$a_{Nw} = N_w D_m^{\ 5}$ vs. D_m



$$a_{Nw} = N_w D_m^5$$

a_{Nw} is independent of Dm

Wide range of a_{Nw}