GPM/DPR Level-3

Algorithm Theoretical Basis Document

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The Level 3 DPR product provides space-time statistics of the level 2 DPR results. High and low spatial resolution grids are defined such that the high-resolution grid is $0.25^0 \times 0.25^0$ (lat×lon) while the low-resolution grid is $5^0 \times 5^0$. For the variables defined on the low-resolution grid, the statistics include mean, standard deviation, counts and histogram. For variables defined on the high-resolution grid, the same statistics are computed with the exception of a histogram, which is omitted.

The level 3 code is written so that the 15 or 16 orbits of level 2 DPR data produced daily can optionally be processed in two runs, where one output file contains statistics from the ascending orbital passes while the other file contains statistics from the descending passes. Since all orbits for the day are processed in each run, there is no need for intermediate files. What is produced are two daily level 3 HDF files. Nominally, the standard level 3 product will be obtained by processing the twice-daily HDF files over a calendar month; however, this is not required. In particular, output products can be generated from any set of daily HDF files. It should be noted that the daily files will contain a mean square statistic rather than the standard deviation. For the monthly (or multi-day) file, however, the mean square statistic will be replaced with the standard deviation.

Six basic variable types have been identified. Type 1 variables are range-dependent but angle independent; type 2 variables are range independent but angle dependent; type 3 are both range and angle independent. Two additional types have been defined to store observation counts (type 4) and time dependent variables (type 5). A final type (type 6) is used to store products suitable for a general user.

Instead of separate level 3 products for the Ku-, Ka-, KaHS and dual-frequency derived products, a decision was made to have a single level 3 output (and processing code) that would include results from the separate *channels or instruments*. Because of this, an additional index, *chn* (1–5) or *inst* (1–4) is needed to store most of the results. For example, to store the mean value of the attenuation-corrected radar reflectivity factor, zFactorCorrected, at Ku-band (inst=1) on the low-resolution grid (G1), we have

`G1%zFactorCorrected%mean(`*ltL,lnL,inst=1,hgt,rt,st*`)`

Note that this quantity is a function of the low-resolution latitude and longitude (ltL, lnL) at a height above the ellipsoid specified by index *hgt* for raintype index *rt* and surface type index *st*. (Note that the statistics for all rain types and all surface types are obtained by setting rt=3 and st=3.) The Ka-band and KaHS-band mean dBZ data are stored in the inst=2 and inst=3 locations, respectively. *In version 2 of the* *code, inst=4 and chn=5 are used to store the Ku-band data in the inner swath so that the Ku- and Ka-band products can be compared over the identical swaths (see discussion below).*

Since attenuation correction may be different when both frequencies are used (dpr), a second variable type is needed to store these data:

`G1%zFactorCorrectedDPR%mean(`*ltL,lnL,inst=1,hgt,rt,st*`)`

In other words, `G1%zFactorCorrected%mean(`*ltL,lnL,inst=1,hgt,rt,st*`)` is use to store the mean Ku-band reflectivity factor using single-frequency attenuation correction methods while `G1%zFactorCorrectedDPR%mean(`*ltL,lnL,inst=1,hgt,rt,st*`)` is used to store the mean Ku-band reflectivity factor using dual-frequency attenuation correction methods.

The need to distinguish between the 'inst' and 'chn' indices arises from the fact that some variables such as the radar reflectivity factor, dBZ, are inherently instrument (inst) derived quantities whereas other variables such as rain rate are geophysical quantities that can be derived either from single- or dual-frequency data. Because of this difference, there can be a rain rate derived from the dual-frequency data but not a radar reflectivity factor derived from dual-frequency data. Variables that use 'chn' have a dual-frequency output (chn=4) whereas variables that use 'inst' (instrument) do not. It should also be noted that the different values of inst or chn correspond to data taken from different swaths. The standard Ku products are derived from the full or normal swath (NS) of 245 km composed of 49 fields of view (chn=inst=1); Ka products from the inner or matched swath (MS) of 125 km (chn=inst=2), KaHS from the interleaved swath (chn=inst=3) (high sensitivity or HS swath) of slightly less than 125 km and the dual-frequency products from the matched swath (chn=4). For version 2 of the code, products have been added (inst=4 or chn=5) that corresponds to Ku-band data taken from the matched/inner swath. These KuMS products can be compared directly with the KaMS products since the data are extracted from the matched swath data. These 5 data swaths, using index chn, are designated by the following: KuNS, KaMS, KaHS, dprMS, KuMS. The 4 'instrument' swaths, using index inst, include all but the dprMS swath: KuNS, KaMS, KaHS, KuMS.

As shown by these examples, the results are stored in a two-tiered structure. The data are stored either in a low-resolution (G1) or high-resolution (G2) structure. A second structure is used to store the statistics for each variable. Each low-resolution variable has associated with it a mean, standard deviation, count and histogram. This holds for the high-resolution variable as well except for the histogram. Another difference between the low and high-resolution grids is that the low-resolution variables are conditioned on surface type (st) whereas the high-resolution variables are not.

Type-1A variables (height/range dependent):

zFactorCorrected, zFactorCorrectedDPR, epsilonDPR, zFactorMeasured

Low-resolution structure for ***zFactorCorrected*** variable (others are the same):

`G1%zFactorCorrected%count(`*ltL,lnL,inst,hgt,rt,st*`)`

`G1%zFactorCorrected%mean`(*ltL,lnL,inst,hgt,rt,st*)

`G1%zFactorCorrected%stdev`(*ltL,lnL,inst,hgt,rt,st*)

`G1%zFactorCorrected%hist`(*ltL,lnL,inst,hgt,rt,st,nbin*)

High-resolution structure for ***zFactorCorrected*** variable (others are the same):

`G2%zFactorCorrected%count`(*ltH,lnH,inst,hgt,rt*)

`G2%zFactorCorrected%mean`(*ltH,lnH,inst,hgt,rt*)

`G2%zFactorCorrected%stdev`(*ltH,lnH,inst,hgt,rt*)

Indices:

ltL: 28 (-70 to 70 deg; 5 deg grid)

lnL: 72 (-180 to 180 deg; 5 deg grid)

ltH: 536 (-67 to 67 deg; 0.25 deg grid)

lnH: 1440 (-180 to 180 deg; 0.25 deg grid)

inst: 4 {KuNS, KaMS, KaHS, KuMS}

hgt: 5 {2 km, 4 km, 6 km, 10 km, 15 km}

rt: 3 {stratiform, convective, all}

st: 3 {ocean, land, all}

nbin: 30 (number of bins in histogram)

Note that all quantities are conditioned on the presence of rain so that the counts refer to rain counts, the mean refers to the conditional (sample) mean, stdev refers to the (sample) standard deviation of all observations with non-zero rain.

Type-1B variables (height/range dependent):

precipRate, rainRate, snowRate, mixedPhRate

Low-resolution structure for ***precipRate*** variable:

`G1%precipRate%count`(*ltL,lnL,chn,hgt,rt,st*)

`G1%precipRate%mean`(*ltL,lnL,chn,hgt,rt,st*)

`G1%precipRate%stdev`(*ltL,lnL,chn,hgt,rt,st*)

`G1%precipRate%hist`(*ltL,lnL,chn,hgt,rt,st,nbin*)

High-resolution structure for ***precipRate*** variable:

`G2%precipRate%count` (*ltH,lnH,chn,hgt,rt*)

`G2%precipRate%mean` (*ltH,lnH,chn,hgt,rt*)

`G2%precipRate%stdev` (*ltH,lnH,chn,hgt,rt*)

Indices (other indices are the same as above):

chn: 5 {KuNS, KaMS, KaHS, dprMS, KuMS}

For these variables, rain rates, precipitation rates, etc., the conversion to unconditioned means, standard deviations, etc., can be done by computing the probability of rain, as described in the section on Type 4 variables (observation counts).

Type-1C variables (height/range dependent):

dm, dBNw

Low-resolution structure for ***dm*** variable (and ***dBNw***):

`G1%dm%count` (*ltL,lnL,hgt,rt,st*)

`G1%dm%mean` (*ltL,lnL,hgt,rt,st*)

`G1%dm%stdev` (*ltL,lnL,hgt,rt,st*)

`G1%dm%histogram` (*ltL,lnL,hgt,rt,st,nbin*)

High-resolution structure for ***dm*** variable (and ***dBNw***):

`G2%dm%count` (*ltH,lnH,hgt,rt*)

`G2%dm%mean` (*ltH,lnH,hgt,rt*)

`G2%dm%stdev` (*ltH,lnH,rt,hgt,nbin*)

Type-2 variables (angle dependent, height independent):

piaSRT, piaSRTdpr, piaFinal, piaFinalDPR, piaFinalSubset, piaFinalDPRSubset

Low-resolution structure for ***piaSRT*** variable:

`G1%piaSRT%count` (*ltL,lnL,inst,ang,rt,st*)

`G1%piaSRT%mean` (*ltL,lnL,inst,ang,rt,st*)

`G1%piaSRT%stdev(`*ltL,lnL,inst,ang,rt,st*`)`

`G1%piaSRT%histogram(`*ltL,lnL,inst,ang,rt,st,nbin*`)`

High-resolution structure for ***piaSRT*** variable:

`G2%piaSRT%count(`*ltH,lnH,inst,ang,rt*`)`

`G2%piaSRT%mean(`*ltH,lnH,inst,ang,rt*`)`

`G2%piaSRT%stdev(`*ltH,lnH,inst,ang,rt*`)`

The *ang* index is taken to be *7.* At Ku-band all 7 locations are used whereas only 4 are needed for the Ka and KaHS data. Taking a 4 angle-bin increment (3°), beginning with nadir (0^0) gives the following incidence angles: 0^0, ±3°, ±6°, ±9° for Ku- and Ka-band. For Ku-band, this series is continued with samples at ±12°, ±15°, ±18°. Specifically, for the Ku-band (inst=1):

ang={1, 2,..,7} $\Rightarrow$ angle bins {25, (21,29), (17,33), (13,37), (9,41), (4,45), (1,49)}

In other words, ang index 1 is used to store the nadir data at Ku-band, index 2 is used to store Ku-band data from angle bins 21 and 29 ($4*0.75^0=3^0$), index 3 to store data from angle bins 17 and 33 ($8*0.75^0=6^0$), and so on, where index 7 is used to store data from the last angle of the outer swath ($24*0.75^0=18^0$).

For the Ka-band channel (inst=2) using the same increment (4 angle bins or 3°) but noting that the center angle bin (counting from 1 to 25) is 13 then:

ang={1, 2, 3, 4} $\Rightarrow$ angle bins {13, (9,17), (5,21), (1,25)}

For the KaHS channel (inst=3), we begin with the two beams closest to nadir so that:

ang={1, 2, 3, 4} $\Rightarrow$ angle bins {(12,13), (8,17), (4,21), (1,24)}

which correspond to incidence angles of ±0.375°, ±3.375°, ±6.375° and ±8.625°.

It will often be the case that the PIA from the SRT is considered unreliable. Since the statistics of ***piaSRT*** and ***piaSRTdpr*** are taken only for those data that are considered marginally reliable or reliable, then comparisons between the PIA(SRT) and PIA(Final) statistics will be taken over different sets of data. To restrict the statistics of PIA(final) to only those cases for which the ***piaSRT*** and ***piaSRTdpr*** are reliable or marginally reliable, we introduce the subsetted statistics: ***piaFinalSubset*** and ***piaFinalDPRSubset***.

Type-3 variables (height and angle independent):

heightBB, heightBBnadir, BBwidth, BBwidthnadir, heightStormTop, precipRateNearSurface, precipRateESurface, precipRateESurface2, precipRateAve24, zFactorCorrectedNearSurface, zFactorCorrectedESurface, zFactorCorrectedNearSurfaceDPR, zFactorCorrectedESurfaceDPR,snowRateNearSurface, mixedPhNearSurface, epsilon

The low-resolution structure for ***heightBB*** variable (and others) is:

`G1%heightBB%count`(*ltL,lnL,chn,rt,st*)

`G1%heightBB%mean`(*ltL,lnL,chn,rt,st*)

`G1%heightBB%stdev`(*ltL,lnL,chn,rt,st*)

`G1%heightBB%histogram`(*ltL,lnL,chn,rt,st,nbin*)

The high-resolution structure for ***heightBB*** variable (and others) is:

`G2%heightBB%count`(*ltH,lnH,chn,rt*)

`G2%heightBB%mean`(*ltH,lnH,chn,rt*)

`G2%heightBB%stdev`(*ltH,lnH,chn,rt*)

Type-4 variables, Total number of observations

ObservationCounts (This structure has 4 elements to store the observation counts with respect to total, local time, angle/pia, and shallow rain)

`G1%ObservationCounts%total`(*ltL,lnL,inst,st*)

`G1%ObservationCounts%localTime`(*ltL,lnL,inst,time,st*)

`G1%ObservationCounts%pia`(*ltL,lnL,inst,ang,st*)

`G1%ObservationCounts%shallowRain`(*ltL,lnL,inst,st*)

where the index *time* (=24) represents local time binned by hour. The high-resolution ObservationCounts structure is the same except the surface type *st* is omitted and the element 'localTime' is not computed.

`G2%ObservationCounts%total`(*ltH,lnH,inst*)

`G2%ObservationCounts%pia`(*ltH,lnH,inst,ang*)

`G2%ObservationCounts%shallowRain`(*ltH,lnH,inst,st*)

Note that ***ObservationCounts%total*** is equal to the number of observations at a particular lat/lon box for each channel and, in the case of the low-resolution grid, for each surface type. The probability of rain at a particular height level, for a particular rain type and surface type, over the low-resolution grid is computed by:

$$\text{Probability of Rain}(ltL,lnL,hgt,chn,rt,st) = \frac{\text{G1\%precipRate\%count}(ltL,lnL,hgt,chn,rt,st)}{\text{G1\%ObservationCounts\%total}(ltL,lnL,hgt,chn,rt,st)}$$

Note that all rain types and all surface types are obtained by setting rt=3 and st=3. The unconditioned mean can be calculated from the conditioned mean by multiplying by the probability of rain. The unconditioned standard deviation can also be computed from the conditional mean, conditional standard deviation and the probability of rain.

Type-5 variables (time-dependent rain rate):

precipRateLocalTime

Low-resolution structure for ***precipRateLocalTime*** variable:

`G1%precipRateLocalTime%count`(*ltL,lnL,chn,time,st*)

`G1%precipRateLocalTime%mean`(*ltL,lnL,chn,time,st*)

`G1%precipRateLocalTime%stdev`(*ltL,lnL,chn,time,st*)

where all indices are defined as before and where

time: 24 {corresponding to hourly grid of the local time}

Note that a height index or rain type index is not included since only the near-surface precipRate is used for this product. Both stratiform and convective rain are included – i.e., no rain type classification is used.

A high-resolution grid for these variables has not been defined.

Type-6 variables (general user products)

precipRateNearSurfaceUnconditional, PrecipProbabilityNearSurface

Since most users will not need the detailed statistics described above, a subset of the mean, near-surface unconditional rain rate is defined which is independent of rain type or surface type, i.e., all rain types and surface types are included.

`G1%precipRateNearSurfaceUnconditional`(*ltL,lnL,chn*)

along with the high-resolution structure:

`G2%precipRateNearSurfaceUnconditional`(*ltH,lnH,chn*)

Since these rain rates will be unconditional, there is no need for a separate count variable. However, the user might want a rain probability:

`G1%precipProbabilityNearSurface`(*ltL,lnL,chn*)

with the corresponding high-resolution structure:

`G2%precipProbabilityNearSurface(`*ltH,lnH,chn*`)`

Definition of Variables (see level 2 documentation for detailed definitions)

(Unless otherwise indicated, the variables below are such that the mean and standard deviations are 4-byte real, the counts and histograms are 4-byte integers. With the exception of ObservationCounts and precipRateNearSurfaceUnconditional, all statistics are conditioned on the presence of precipitation. Unless otherwise noted, all variables are defined on both low and high-resolution grids.)

dBN_w: 10 log10 of the particle number concentration (m^{-3}) [solver]

dm: mass-weighted diameter (mm) [solver]

epsilon: dimensionless scale factor on α in $k=\alpha Z^{\beta}$ (where k is the specific attenuation in dB/km) [solver]

epsilonDPR: same as above except height dependent using dual-freq data [solver]

heightBB: height from ellipsoid to 'bright-band' (m) [classification]

heightBBnadir: height from ellipsoid to 'bright-band' for nadir incidence (m) [classification]

heightStormTop: height from ellipsoid to storm top (m) [preparation]

mixedPhRateNearSurface: precip rate of mixed phase particles near surface (mm/h) [solver]

ObservationCount%localTime: total number of observations categorized into local hour. Note that this variable is only computed on the low-resolution grid.

ObservationCount%pia: total number of observations categorized into incidence angle

ObservationCount%shallowRain: number of observations of shallow rain [classification]

ObservationCount%total: total number of observations

piaSRT: path-integrated attenuation (dB), obtained from single-freq methods [SRT]

piaSRTdpr: path-integrated attenuation (dB), obtained from dual-freq method [SRT]

piaFinal: path-integrated attenuation (dB), obtained from single-freq methods [solver]

piaFinaldpr: path-integrated attenuation (dB), obtained from dual-freq method [solver]

piaFinalSubset: path-integrated attenuation (dB), obtained from single-freq methods using only those observations for which the SRT-derived pia is considered reliable or marginally reliable [SRT, solver]

piaFinalSubsetDPR: path-integrated attenuation (dB), obtained from dual-freq method using only those observations for which the SRT-derived pia is considered reliable or marginally reliable [SRT, solver]

PrecipProbabilityNearSurface: probability of rain near surface [preparation]

precipRate: height-dependent precipitation rate (mm/h). Note that all 'precipRate' variables include all types of precipitation [solver]

precipRateAve24: average precipitation rate (mm/h) between 2-4 km above ellipsoid [solver]

precipRateLocalTime: near-surface precip rate (mm/h) categorized into local hour; low-resolution only [solver]

precipRateESurface: estimated precip rate at surface (mm/h) [solver]

precipRateESurface2: estimated precip rate at surface (mm/h), using a statistical approach [solver]

precipRateNearSurface: precip rate(mm/h) near surface [solver]

precipRateNearSurfaceUnconditional : unconditional rain rate (mm/h) near surface, low-resolution only [solver]

precipProbabilityNearSurface: probability of precipitation near surface, low-resolution only [solver]

rainRate: height-dependent rain rate (mm/h) [solver].

snowRate: height-dependent snow rate (mm/h) [solver].

snowRateNearSurface: snow rate near surface (mm/h) [solver]

stormHeight: height of maximum detectable echo from ellipsoid (m) [preparation]

widthBB: width of bright-band (m) [classification]

widthBBnadir: width of bright-band (m) at nadir incidence [classification]

zFactorCorrected: height-dependent radar reflectivity factor (mm^6/m^3) in dB, using single-freq attenuation correction [solver]

zFactorCorrectedDPR: height-dependent radar reflectivity factor (mm^6/m^3) in dB, using dual-freq attenuation correction [solver]

zFactorCorrectedESurface: estimated at-surface radar reflectivity factor (mm^6/m^3) in dB, using single-freq attenuation correction [solver]

zFactorCorrectedESurfaceDPR: estimated at-surface radar reflectivity factor (mm^6/m^3) in dB, using dual-freq attenuation correction [solver]

zFactorCorrectedNearSurface: near-surface radar reflectivity factor (mm^6/m^3) in dB, using single-freq attenuation correction [solver]

zFactorCorrectedNearSurfaceDPR: near-surface radar reflectivity factor (mm^6/m^3) in dB, using dual-freq attenuation correction [solver]

zFactorMeasured: height-dependent measured radar reflectivity factor (mm^6/m^3) in dB [preparation]

```
 ! Histogram categories
 real, dimension(nhbin+1), parameter    ::      &

   cat_rain = [ 0.01,    &    ! mm/h  (logarithmic steps)
        0.10,   0.13,   0.17,   0.23,   0.30,   0.40,    &
        0.52,   0.69,   0.91,   1.20,   1.58,   2.08,    &
        2.75,   3.62,   4.77,   6.29,   8.29,  10.92,    &
       14.40,  18.97,  25.00,  32.95,  43.43,  57.24,    &
       75.44,  99.43, 131.04, 172.71, 227.63, 300.00 ],  &

   cat_Z = [ 0.01,   &        ! dBZ
        6.0,    8.0,   10.0,   12.0,   14.0,   16.0,     &
       18.0,   20.0,   22.0,   24.0,   26.0,   28.0,     &
       30.0,   32.0,   34.0,   36.0,   38.0,   40.0,     &
       42.0,   44.0,   46.0,   48.0,   50.0,   52.0,     &
       54.0,   56.0,   58.0,   60.0,   62.0,   64.0 ],   &

   cat_integratedWater = [ 0.0,   &       ! kg/m^2
      200.0,  400.0,  600.0,  800.0, 1000.0, 1200.0,     &
     1400.0, 1600.0, 1800.0, 2000.0, 2200.0, 2400.0,     &
     2600.0, 2800.0, 3000.0, 3200.0, 3400.0, 3600.0,     &
     3800.0, 4000.0, 4200.0, 4400.0, 4600.0, 4800.0,     &
     5000.0, 5200.0, 5400.0, 5600.0, 5800.0, 6000.0 ],   &

   cat_bbhgt = [ 10.0,  &     ! meters
      250.0,  500.0,  750.0, 1000.0, 1250.0, 1500.0,     &
```

```
    1750.0, 2000.0, 2250.0, 2500.0, 2750.0, 3000.0,     &
    3250.0, 3500.0, 3750.0, 4000.0, 4250.0, 4500.0,     &
    4750.0, 5000.0, 5250.0, 5500.0, 5750.0, 6000.0,     &
    6250.0, 6500.0, 6750.0, 7000.0, 7500.0,20000.0 ],   &

cat_bbwdth = [ 0.0,  &     ! meters
     125.0,  250.0,  375.0,  500.0,  625.0,  750.0,     &
     875.0, 1000.0, 1125.0, 1250.0, 1375.0, 1500.0,     &
    1625.0, 1750.0, 1875.0, 2000.0, 2125.0, 2250.0,     &
    2375.0, 2500.0, 2625.0, 2750.0, 2875.0, 3000.0,     &
    3125.0, 3250.0, 3375.0, 3500.0, 3625.0, 3750.0 ],   &

cat_stormh = 1000.0*[ 0.01, &     ! km  (convert m > km)
       0.5,    1.0,    1.5,    2.0,    2.5,    3.0,     &
       3.5,    4.0,    4.5,    5.0,    5.5,    6.0,     &
       6.5,    7.0,    7.5,    8.0,    8.5,    9.0,     &
       9.5,   10.0,   10.5,   11.0,   11.5,   12.0,     &
      12.5,   13.0,   14.0,   15.0,   16.0,   20.0 ],   &

cat_epsilon = [ 0.0, &
       0.1,    0.2,    0.3,    0.4,    0.5,    0.6,     &
       0.7,    0.8,    0.9,    1.0,    1.1,    1.2,     &
       1.3,    1.4,    1.5,    1.6,    1.7,    1.8,     &
       1.9,    2.0,    2.1,    2.2,    2.3,    2.4,     &
       2.5,    2.6,    2.7,    2.8,    2.9,    3.0 ],   &

cat_nubf = [ 1.0, &
       1.05,   1.1,    1.15,   1.2,    1.25,   1.3,     &
```

```
     1.35,   1.4,    1.45,   1.5,    1.55,   1.6,     &
     1.65,   1.7,    1.75,   1.8,    1.85,   1.9,     &
     1.95,   2.0,    2.1,    2.2,    2.3,    2.4,     &
     2.5,    2.6,    2.7,    2.8,    2.9,    3.0 ],   &

cat_pia = [ 0.01, &
     0.1,    0.2,    0.3,    0.4,    0.5,    0.6,     &
     0.8,    1.0,    1.2,    1.4,    1.6,    1.8,     &
     2.0,    2.5,    3.0,    3.5,    4.0,    4.5,     &
     5.0,    5.5,    6.0,    7.0,    8.0,    9.0,     &
    10.0,   15.0,   20.0,   25.0,   30.0,  100.0 ],   &

cat_dBNw = [ 0.1,  &
     1.0,    2.0,    4.0,    6.0,    8.0,   10.0,     &
    12.0,   14.0,   16.0,   18.0,   20.0,   22.0,     &
    24.0,   26.0,   28.0,   30.0,   32.0,   34.0,     &
    36.0,   38.0,   40.0,   42.0,   44.0,   46.0,     &
    48.0,   50.0,   52.0,   54.0,   56.0,   60.0 ],   &

cat_Dm = [ 0.1,   &       ! mm
     0.2,    0.3,    0.4,    0.5,    0.6,    0.7,     &
     0.8,    0.9,    1.0,    1.1,    1.2,    1.3,     &
     1.4,    1.5,    1.6,    1.7,    1.8,    1.9,     &
     2.0,    2.1,    2.2,    2.3,    2.4,    2.5,     &
     2.6,    2.7,    2.8,    2.9,    3.0,    4.0 ]
```